ON THE METRIC OF WAVES INTERACTIONS

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1. PROCEDURE OF WAVES MEASUREMENT WITH THE HELP OF WAVES

The objects with wave character in the world surrounding us are universal. The wave nature is common as for the standards of time and length, so and the objects measured with their help, which are consisting from elementary particles. It is enough to recollect that, as the standard of time - second is accepted certain number of periods of atomic oscillations of cesium (Cs), and as the standard of length - certain number of wavelengths of atomic radiation of krypton (Kr). Otherwise, the temporary and spatial physical measurements are the operations of comparison with characteristics of waves: with period T and with length of a wave λ. But if all surrounding us have the properties of wave, all this may be described from positions common for all systems having waves character. The idea about a construction of the metric system, based on properties of waves, follows from here. Or else, the speech goes about *the measurements and relations between results of these measurements in different systems of reference, if the objects of measurement and the measuring instruments have a wave character simultaneously*. This problem is important when researching the interactions between waves. The determination of metric the waves interactions is equivalent to understanding, how waves "see" or "perceive" each other.

For simplicity, in the beginning we shall suppose that, the instruments and the objects of supervision are described by harmonic functions. Let us assume that, the standing wave is accepted as the instrument and is described by expression:

$$a = A\cos(-kx)\cos(\omega t) . \qquad (1)$$

By setting such wave, we thus set the metric, namely:

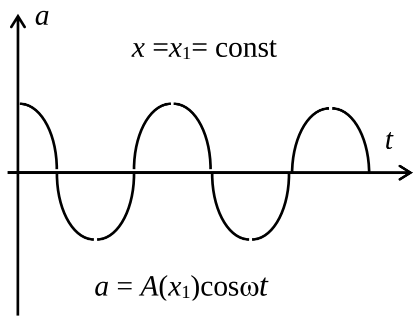
- the direction of the axis x - coincides with direction of propagation of the wave;
- the spatial scale - is defined by length of a wave

$$\lambda = \frac{2\pi}{k};$$

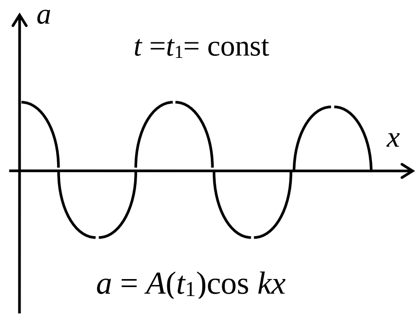
- temporary scale - is defined by period of a wave

$$T = \frac{2\pi}{\omega}.$$

Otherwise, the standing wave executes the same role by a natural mode, as the rulers and the clocks in the special theory of relativity.



a)



b)

Fig. 1. A frame, determined by standing wave: (1) a) - in a fixed point; b) - in a fixed instant.

Thus $\cos\omega t$ defines an instantaneous value of wave amplitude a, in the fixed point, with a coordinate x_1 (figure 1a). It is possible to say that this term sets the rhythm of time or serves as a chronometer in the examined point.

The term $\cos kx$ sets the dependence of oscillations amplitude from a co-ordinate (figure 1b), hence, the position of points with an identical phase of maximums, determines the spatial scale. The question on physical sense of amplitude A will be discussed after.

May appear the question: how we measure x and t, if the wave (1) himself serves as system of co-ordinates. It is important the physical existence of wave (1) as a standard. Such as exists the meter (or ft), the length of which is not measured before using.

The wave-object, "resting" in system (1), will be described by similar expression:

$$a_0 = A\cos(-k_0 x)\cos(\omega_0 t). \qquad (2)$$

The measurement of its length in system of co-ordinates (1) consists in the ratio determination between lengths of object-wave and scale-wave

$$n = \lambda_0 / \lambda, \qquad (3)$$

Similarly, the period measurement of object-wave consists in ratio determination between the periods of the object-wave and scale-wave:

$$n = T_0 / T, \qquad (4)$$

The question about signals, "used" by the waves, for executing the discussed measurements, is deprived of sense. The waves are not beside with each other, but are exist in the same medium simultaneously. They are imposed accordingly to a principle of a superposition.

The wave-object (2) can be decomposed into two travelling waves, which run in opposite directions, of a kind:

$$a_{01} = \frac{A}{2}\cos(\omega_0 t - k_0 x) \qquad (5)$$

$$a_{02} = \frac{A}{2}\cos(\omega_0 t + k_0 x), \qquad (6)$$

It is possible to be convinced by immediate substitution in that: $a_0 = a_{01}+a_{02}$. In common case, in expressions (5) and (6) - frequencies and the wave numbers can be different:

$$a_{01} = \frac{A}{2}\cos(\omega_{01} t - k_{01} x) \qquad (7)$$

$$a_{02} = \frac{A}{2}\cos(\omega_0 t + k_0 x), \qquad (8)$$

where $\omega_{01} \neq \omega_0$ and $k_{01} \neq k_0$. Then the wave-object will be described by the formula:

$$a_0 = a_{01} + a_{02} = \\ = A_0 \cos\left(\frac{\omega_{01}-\omega_0}{2}t - \frac{k_0+k_{01}}{2}x\right)\times, \qquad (9) \\ \times\cos\left(\frac{\omega_{01}+\omega_0}{2}t - \frac{k_0-k_{01}}{2}x\right)$$

The expression (9) describes a quasi-standing wave, which moves relatively of laboratory frame (1). The value

$$\alpha = \frac{\omega_{01}-\omega_0}{2}t \qquad (10)$$

in the first factor determines the shift of nodes and loops of object-wave along a spatial co-ordinate. Really, in the formula (2) the first factor describes motionless periodic function in the correspondence with figure 1b. If to add a time-dependent component to argument of this factor, then with the course of time this function will be moved along *x*. Of it, it is possible to be convinced by substituting numerical values.

Similarly value

$$\theta = \frac{k_0 - k_{01}}{2}x$$

in second factor determines the delay or the shift along the temporary co-ordinate. Physically it means that, the phase of a standing wave varies on the defined value with each period. We shall reduce the spatial shift to the length dimension. For this purpose, we shall divide formula (10) on a factor at *x*, (that is into a measure of length):

$$\Delta x = \frac{\omega_{01}-\omega_0}{k_0+k_{01}}\Delta t. \qquad (11)$$

Formula (11) expresses the object-wave shift along co-ordinate *x* during time Δt. In particular, if $\Delta t = T$ (where *T* is the period), then Δx will be equal to shifting during period. Hence, the velocity of transition of object-wave is

$$v_0 = \frac{\Delta x}{\Delta t} = \frac{\omega_{01}-\omega_0}{k_0+k_{01}}. \qquad (12)$$

In view of that

$$\omega = \frac{2\pi}{T}, \; k = \frac{2\pi}{\lambda}$$

and $$\frac{\lambda}{T} = \frac{\omega}{k} = c,$$

expression (12) is possible to copy such as:

$$v_0 = \frac{\lambda_0\lambda_{01}}{T_0T_{01}}\frac{T_0-T_{01}}{\lambda_{01}+\lambda_0} = c^2\frac{T_0-T_{01}}{\lambda_{01}+\lambda_0} = c^2\frac{k_0-k_{01}}{\omega_0+\omega_{01}}.$$

We shall remind that, v_0 is the velocity of transition of points with an identical phase, for example, of maximums of quasi-standing wave.

The difference between expressions (2) and (9) consists in that that, (2) describes the wave-object motionless concerning a system of reference (1), but (9) describes wave-object moving with velocity v_0.

Thus, a modification of velocity or acceleration is connected with reorganization of components of standing wave this requires additional efforts. Therefore acceleration may be detected and requires application of exterior force.

The velocity v_0, defined by expression (9), is not linked with movement of the continuum or relatively him. *But, in lack of other tools except waves, only she can characterize transition of objects.*

If $\omega_0 << \omega_{01}$, velocity $v_0 \to c$, and at $\omega_0 >> \omega_{01}$, velocity, $v_0 \to -c$. That means the quasi standing wave is transformed in running wave. When $\omega_0 = \omega_{01}$, velocity $v_0 = 0$. This corresponds to the standing wave. Thus, the absolute value of velocity v can varies from 0 up to c. From here we can make conclusion that *velocity of propagation of perturbation c serves as a natural limit of velocity of transition of objects.*

2. THE THEOREM ABOUT INVARIANCE OF MAXIMUM VELOCITY OF PERTURBATION PROPAGATION RELATIVELY TO A SYSTEMS OF REFERENCE

Just as the wave-object (2) was decomposed on components (5) and (6), also it is possible to decompose the wave-instrument (1) in components:

$$a_1 = \frac{A}{2}\cos(\omega t - kx) \qquad (13)$$

$$a_2 = \frac{A}{2}\cos(\omega t + kx)\,, \qquad (14)$$

If in the components (13) and (14) the frequencies and the wave numbers differ, it means that:

$$a_1' = \frac{A}{2}\cos(\omega_1 t - k_1 x)$$

$$a_2' = \frac{A}{2}\cos(\omega t + kx)\,,$$

we shall receive the new wave-instrument:

$$a' = A\cos\left(\frac{\omega_1 - \omega}{2}t - \frac{k + k_1}{2}x\right)\times \times\cos\left(\frac{\omega_1 + \omega}{2}t - \frac{k - k_1}{2}x\right), \qquad (15)$$

which moves relatively (1) with a velocity

$$v = \frac{\omega_1 - \omega}{k + k_1} = c^2\,\frac{k - k_1}{\omega + \omega_1} = c^2\,\frac{T - T_1}{\lambda_1 + \lambda}\,. \qquad (16)$$

When the velocities of wave-object (9) and wave-instrument (15) are identical $v_0 = v$, the wave-object (9), moving relatively of a non-stroked frame of reference (1), will be resting in a stroked frame of reference (15). It means, that (9) and (15) are in rapport as well as (1) and (2). The wave-object (9), in system (15), will be described by expression for a standing wave similar to (2):

$$a_0' = A_0\cos(-k_0' x')\cos(\omega_0' t')\,. \qquad (17)$$

The stroked frame (15) "from the own point of view" is motionless, hence, it will be defined by expression similar to (1):

$$a' = A\cos(-k' x')\cos(\omega' t') \qquad (18)$$

As between (2) and (1) the relations (3) and (4) exists in a stroked system, between wave-object (17) and wave-instrument (18), will take place:

$$\lambda_0' / \lambda' = n \qquad (3')$$

$$T_0' / T' = n \qquad (4')$$

The expressions (3') and (4') define the act of measurement, as well as (3) and (4). As the result in both cases is a same, obviously, it is impossible to decide which of systems, i.e. non-stroked (1) or stroked (18), "is true motionless" relatively to spatial - temporary continuum, even if the last is the real carrier of waves. Thus, the relativity principle or, otherwise, the principle of equivalence of all systems of reference, in model, offered by us, is kept.

We shall copy (9) with the account (12):

$$a_0 = A_0\cos\left(\frac{k + k_1}{2}(vt - x)\right)\times \times\cos\left(\frac{\omega + \omega_1}{2}(t - \frac{v}{c^2}x)\right). \qquad (19)$$

The value (vt-x) in (19) represents the instantaneous co-ordinate of wave-object, moving in non-stroked frame (1) with velocity v (we shall compare it with a motionless wave-object (2)). But, if the expression, describing a co-ordinate, varies, the expression for length of a segment will vary in the same way. Hence the length of moving object-wave is already will be $\lambda'=\lambda-vT$, and its wave number is:

$$k' = \frac{2\pi}{\lambda - vT}. \qquad (20)$$

From similar reasoning for frequency, we shall receive:

$$\omega' = \frac{2\pi}{T - \frac{v}{c^2}\lambda}. \qquad (21)$$

The ratio of circular frequency ω to a wave number k is equal to a velocity of propagation of travelling waves or propagation velocity of perturbation c.

We shall demonstrate the **theorem that the velocity of propagation of perturbation do not depend from the choice of reference system**, it is equivalent to:

$$\omega/k = \omega'/k' = c.$$

By using (20) and (21) we shall receive:

$$c' = \frac{\omega'}{k'} = \frac{2\pi / (T - \frac{v}{c^2}\lambda)}{2\pi / (\lambda - vT)} = c^2 \frac{\lambda - vT}{c^2 T - v\lambda}.$$

In view of expression (16),

$$c' = \frac{\lambda - Tc^2\left(\frac{T - T_1}{\lambda + \lambda_1}\right)}{T - \lambda\left(\frac{T - T_1}{\lambda + \lambda_1}\right)} = \frac{\lambda(\lambda + \lambda_1) - Tc^2(T - T_1)}{T(\lambda + \lambda_1) - \lambda(T - T_1)}.$$

Taking into account that, $\lambda/T = c$,

$$c' = \frac{\lambda(\lambda + \lambda_1) - \lambda c(T - T_1)}{T(\lambda + \lambda_1) - Tc(T - T_1)} = \frac{\lambda}{T} = c.$$

Thus, we showed, that, the velocity of propagation of a travelling wave c does not depend from a choice of reference systems, within the framework of model, in which in quality of frames serve objects, having a wave nature. We shall remind that it is the second of two postulates of the special relativity theory.

3. TRANSFORMATION OF SCALES OF LENGTH AND TIME FOR PASSAGES BETWEEN SYSTEMS OF REFERENCES BASED ON WAVES

As λ and T are the scales, with which the objects from non-stroked system are compared, for process of any extent x and duration t from non-stroked system can to write:

$$x = n\lambda; \qquad t = nT.$$

Similarly for stroked system:

$$x' = n\lambda' \qquad t' = nT'; \qquad . \quad (22)$$

We shall define a relation between stroked and non-stroked values, in other words, the law of transformation of spatial and temporary co-ordinates.

We shall return to the equation (12) and we shall express k_1 through k, v and c.

$$v = \frac{\omega_1 - \omega}{k + k_1} = \frac{c(k_1 - k)}{k + k_1},$$

or, $vk + vk_1 = ck_1 - ck$, from here:

$$k_1 = k\frac{c + v}{c - v}. \qquad (23)$$

Similarly:

$$\omega_1 = \omega\frac{c + v}{c - v}. \qquad (24)$$

We shall introduce (23) and (24) into (19):

$$a_0 = A_O \cos\left[\frac{kc}{c - v}(vt - x)\right] \times \cos\left[\frac{\omega c}{c - v}\left(t - \frac{v}{c^2}x\right)\right]. \qquad (25)$$

We shall compare (25) with (17). Both expressions describe the same value. The expression (25) describes the wave-object in the terms of non-stroked system, and (17) - relatively to stroked system (18).

In both expressions, the arguments of the first cos represent the ratio of length of the same segment of wave-object to an own scale or to the length of a wave. It is dimensionless value or simply a number not depending from a system of reference. Therefore we can equate the appropriate values from (17) and (25), and to receive:

$$x' = \frac{k}{k'}\frac{c}{c-v}(x - vt)\,. \qquad (26)$$

In the correspondence with (20):

$$k^1 = \frac{2\pi}{\lambda - vT} = \frac{n2\pi}{x - vt}\,, \qquad (27)$$

where n is the number of lengths of waves, contained in a distance from a beginning of co-ordinates up to examined point. Similarly, passing from stroked system in non-stroked, it is possible to receive the expression

$$k = \frac{2\pi}{\lambda' - v'T'} = \frac{n2\pi}{x' - v't'}\,. \qquad (28)$$

Here v' - the velocity, with which non-stroked system moves "from the points of view" of stroked system and v - the velocity of stroked system relatively to non-stroked. It is the same value only the direction will be changing on opposite at passage from one system to another. That it is possible to write $v'=-v$, and therefore, (28) will be rewrite:

$$k = \frac{n2\pi}{x' + vt'}\,. \qquad (29)$$

We shall introduce (27) and (29) into (26):

$$x' = \frac{\dfrac{n2\pi}{x'+vt'}}{\dfrac{n2\pi}{x-vt}}\frac{c}{c-v}(x - vt) = \\ = \frac{c}{(c-v)(x'+vt')}(x-vt)^2\,.$$

In the correspondence with (22) $x'/t'=n\lambda'/nT'=c$, hence:

$$x' = \frac{1}{t'}\frac{c}{(c-v)(c+v)}(x-vt)^2\,,$$

that is equivalent:

$$x' = \frac{x - vt}{\sqrt{1 - \dfrac{v^2}{c^2}}} = \frac{x - vt}{\sqrt{1-\beta^2}} = \gamma(x - vt)\,. \qquad (30)$$

where $\beta = \dfrac{v}{c}$ and $\gamma = \dfrac{1}{\sqrt{1-\beta^2}}$.

We shall remind that, x is the object co-ordinate in frame, relatively of which it is moved, and x' - its co-ordinate in system of reference, which is moving together with the object. Thus (30) defines the relation between co-ordinates of two frames, moving relatively each other.

Let's mark a segment, the length of which in a system, in which he rests, is $\Delta x'=x_2'-x_1'$, in correspondence with (30), his length in a system, relatively which he moves, will be

$$\Delta x' = x'_2 - x'_1 = \gamma(x_2 - x_1) = \gamma\Delta x\,. \qquad (31)$$

By doing similar transformations with argument of second cos in (25), we shall receive expression appropriate (26) for the own time:

$$t' = \frac{\omega}{\omega'}\frac{c}{c-v}(t - \frac{v}{c^2}x)\,. \qquad (32)$$

Similarly to the formula (27):

$$\omega' = \frac{2\pi}{T - \dfrac{v}{c^2}\lambda} = \frac{n2\pi}{t - \dfrac{v}{c^2}x}\,, \qquad (33)$$

where, in this case, n - number of periods T of wave-object, past from a beginning of readout up to a considered instant. Accordingly:

$$\omega = \frac{2\pi}{T' - \dfrac{v'}{c^2}\lambda'} = \frac{n2\pi}{t' + \dfrac{v}{c^2}x'}\,. \qquad (34)$$

We shall introduce (33) and (34) into (32):

$$t' = \frac{\dfrac{n2\pi}{t' + \dfrac{v}{c^2}x'}}{\dfrac{n2\pi}{t - \dfrac{v}{c^2}x}}\frac{c}{c-v}(t - \frac{v}{c^2}x) = \\ = \frac{c}{(t' + \dfrac{v}{c^2}x')(c-v)}(t - \frac{v}{c^2}x)^2\,.$$

We shall solve this expression in relation to t', by taking into account that $t'= nT'$, $x'= nl'$ and $\lambda'/T'=c$, we shall receive:

$$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma\left(t - \frac{v}{c^2}x\right),$$

The relation between the results of measurements of interval of time in two systems of reference:

$$\Delta t' = t_2' - t_1' = \gamma(t_2 - t_1) = \gamma\Delta t \ . \qquad (35)$$

Here $\Delta t'$ is the interval of time between two events, happening in the same point x, in the system, where the wave-object is motionless. Δt - same interval, measured in a frame, where the wave-object is moved.

The executed deduction is not connected with specific values of ω and λ, it can be applied to the linear combinations of functions pairs such as (5), (6) and (7), (8). This deduction can also be generalised for two function of the any form, being the superposition of pairs of waves travelling in the opposite directions along l:

$$f_1 = f_1(l + ct)$$
$$f_2 = f_2(l - ct) \ ,$$

if these functions are decomposed in Fourier series or, in a limit, can be described with the help of integrals Fourier. Hence, the described in the present article is also can be applied to a more complex objects than the harmonic waves.

We did not impose any restrictions on amplitude. In particular, it can be radial function of an aspect $A = A(r^{-n})$, where r is radius and n some positive number, then we shall receive centrally symmetric spherical waves, which can describe elementary particles.

CONCLUSIONS

Formulas (31) and (35) - represent Lorentz transformations. Thus, we have shown, that the Lorentz transformation laws take place in a case, when both, objects of measuring and tools, through which these measuring are made, are waves in the same continuum. As the Lorentz transformation laws represent essence of a special relativity theory, from described above follows, that there is no inconsistency between a relativity theory and existence of a material continuum as waves carrier. Moreover, in offered model the postulate about a finiteness of a maximum velocity of signal propagation is transformed in theorem, and becomes clear, why it is impossible to find out the carrier of waves through experiences such as a Michelson-Morley, experiment, that is attempts to spot first derivative of coordinate on time.

Whether has a value a problem about materiality of a spatially - temporal continuum? This problem became to actual connection with attempts to create relativistic propulsions unit. One device of such type is described by Takuya Ishizaka [1].

Within the framework of the existing concepts about lack of a material basis at a spatially - temporal continuum (ether) any device of a similar type can not be implemented, as it contradicts a conservation law of impulse. Simply speaking «it is impossible to be pushed from nothing». The problem gains completely other aspect, if the model, offered by us, is correct. In this case, basically, already there is a basis, from which it is possible to be repelled, and all becomes matter of technique.

Bibliography

1. *Takuya Ishizaka. Inertial force deviation for professional. URL: http://www.sphere.ad.jp/force/relativity/pro/forpro-e.html*