DETERMINATION OF CRITICAL STEADY STATES IN POWER SYSTEMS

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INTRODUCTION

The rigorous estimation of steady-state stability of a Power System requires:

- writing of small-disturbance linearized differential equations for all the elements and control devices;
- writing and solving the characteristic equation and examination of its roots (eigenvalues of the state matrix);

A real eigenvalue corresponds to a non-oscillatory mode. A negative real eigenvalue represents a decaying mode. The larger is its magnitude the faster the decay. A positive real eigenvalue represents aperiodic instability.

Complex eigenvalues occur in conjugate pairs, and each pair corresponds to an oscillatory mode. The real component of the eigenvalues gives the damping, and the imaginary component gives the frequency of oscillation. A negative real part represents a damped oscillation whereas a positive real part represents oscillation of increasing amplitude.

The absolute term of characteristic equation is the determinant of the linearized system differential equations and under certain conditions can coincide with the determinant of power-flow Jacobian [1]. This is valid when the system contains infinite buses and the following conditions are holding true when performing the power flow calculation:

- for generator buses the active power and voltage magnitude are specified;
- loads are specified with the same static
- characteristics as for steady-state stability analysis;
- the slack buses are infinite buses.

Taking into consideration the statement above, the estimation of steady-state aperiodic stability is possible in combination with power-flow calculation. Thus, the power-flow calculation programs can in addition be used for the estimation of steady-state aperiodic stability.

It is known that power-flow equations presented in either forms nodal current or nodal powers contain complex and complex conjugate values. The existence of complex and complex conjugate values makes nodal deviation functions non-derivable relatively to their component variables. Therefore, it is necessary to separate the real and imaginary parts of power flow equations when the iterative methods requiring Taylor-series expansion of nodal deviation functions are utilized.

By separation of real and imaginary parts of power-flow equations it is possible to obtain only solutions for steady states that are physically feasible in power systems. At the same time, if in power system nodes are injected powers that exceed a definite limit, then it is impossible to obtain any real solutions because in this case the steady state physically doesn’t exist. But, the determination of real solution non-existence by the divergence of iterative process is a difficult problem.

In this connection, it is proposed to extend the domain of power flow solutions to the domain of complex numbers. The power-flow equations are written and/or solved in such a way that we can get solutions both for existing and non-existing steady states. The iterative process converges even if steady state doesn’t exist in reality; the solutions are obtained in any case. Analyzing the values of obtained solutions, it is possible to answer the question whether the steady state exists or not. The other advantage is that we can approach the steady-state stability limit point from the domain of non-existing steady states.

1. TWO-NODE STUDY CASE

In this paper the two-node power system case is studied. A general system configuration is shown in fig. 1. Analysis of systems having such simple configurations is extremely useful in understanding basic effects and concepts.

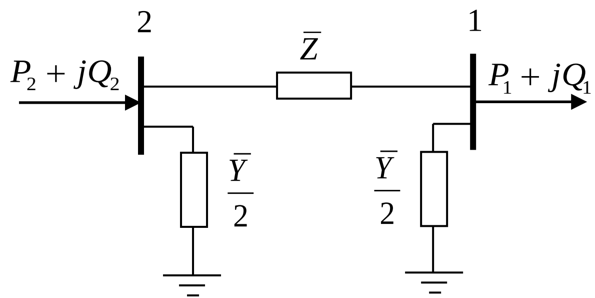

Figure 1. General two-node power system configuration.

1.1. Analytical analysis

In this power system bus 1 is a PQ load bus and bus 2 is a slack bus. For power flow solution the following equation is written:

$$\left(\overline{Y}_{11}\cdot\overline{U}_1+\overline{Y}_{12}\cdot\overline{U}_2\right)\cdot\hat{U}_1=\hat{S}_1 \qquad (1.1)$$

Where:

$\overline{U}_1$, $\overline{U}_2$ – complex voltages in node 1 and 2 respectively;

$\hat{U}_1$, $\hat{U}_2$ – complex conjugates of $\overline{U}_1$ and $\overline{U}_2$ respectively;

$\hat{S}_1$ – complex conjugate of $\overline{S}_1$;

$\overline{Y}_{11}=\frac{1}{\overline{Z}}+\frac{\overline{Y}}{2}$; $\overline{Y}_{12}=-\frac{1}{\overline{Z}}$.

Further all the complex conjugate values will appear using the character ^ over the letter.

After some transformations and using some notations equation (1.1) is written in the following way:

$$\overline{U}_1\cdot\hat{U}_1+\overline{M}\cdot\hat{U}_1=\overline{N}\cdot\hat{S}_1 \qquad (1.2)$$

Where: $\overline{M}=\frac{\overline{Y}_{12}}{\overline{Y}_{11}}\cdot\overline{U}_2$ and $\overline{N}=\frac{1}{\overline{Y}_{11}}$.

Now, we will rewrite equation (1.2) in following way:

$$\begin{aligned}&\left(U_1'+jU_1''\right)\cdot\left(U_1'-jU_1''\right)+\left(M'+jM''\right)\cdot\\&\left(U_1'-jU_1''\right)-\left(N'+jN''\right)\cdot\left(P_1-jQ_1\right)=0\end{aligned} \qquad (1.3)$$

Where: $U_1'=\mathrm{Re}\left(\overline{U}_1\right)$, $U_1''=\mathrm{Im}\left(\overline{U}_1\right)$, $M'=\mathrm{Re}\left(\overline{M}\right)$, $M''=\mathrm{Im}\left(\overline{M}\right)$, $N'=\mathrm{Re}\left(\overline{N}\right)$,. $N''=\mathrm{Im}\left(\overline{N}\right)$.

Performing complex operations on equation (1.3) and separating the real and imaginary parts we will get two real equations:

$$\begin{aligned}&\left(U_1'\right)^2+\left(U_1''\right)^2+M'\cdot U_1'+M''\cdot U_1''-\\&N'\cdot P_1-N''\cdot Q_1=0\end{aligned} \qquad (1.4a)$$

$$M''\cdot U_1'-M'\cdot U_1''+N'\cdot Q_1-N''\cdot P_1=0 \qquad (1.4b)$$

From equation (1.4b) U_1'' is expressed:

$$U_1''=\frac{M''\cdot U_1'+N'\cdot Q_1-N''\cdot P_1}{M'} \qquad (1.5)$$

Substituting (1.5) for U_1'' in equation (1.4a) following quadratic equation is obtained:

$$a\cdot\left(U_1'\right)^2+b\cdot U_1'+c=0 \qquad (1.6)$$

Where:

$$a=\left(\frac{M''}{M'}\right)^2+1 \qquad (1.7a)$$

$$\begin{aligned}&b=M'+\frac{\left(M''\right)^2}{M'}+\frac{2\cdot M''}{\left(M'\right)^2}\cdot(N'\cdot Q_1-\\&N''\cdot P_1)\end{aligned} \qquad (1.7b)$$

$$\begin{aligned}&c=\frac{M''}{M'}\cdot\left(N'\cdot Q_1-N''\cdot P_1\right)-\\&N'\cdot P_1-N''\cdot Q_1+\\&\frac{\left(N'\cdot Q_1\right)^2+\left(N''\cdot P_1\right)^2-2\cdot N'\cdot N''\cdot P_1\cdot Q_1}{\left(M'\right)^2}\end{aligned} \qquad (1.7c)$$

The analytical solution of a quadratic equation is well known. So,

$$U_1'=\frac{-b\pm\sqrt{d}}{2\cdot a} \qquad (1.8)$$

Where,

$$d=b^2-4\cdot a\cdot c \qquad (1.9)$$

Knowing U_1', we easily calculate U_1'' using equation (1.5).

Equation (1.8) gives two solutions for U_1'. One of them corresponds to a stable steady state and the other one to an unstable steady state. In steady-state stability limit point (where $d=0$) we will have two identical solutions.

The maximum power that can be transmitted through line is determined by equating the discriminant of the above quadratic equation (1.6) to zero and solving for P_1. Equating to zero equation (1.9) and substituting instead of a, b and c expressions (1.7a), (1.7b) and (1.7c) the following quadratic equation is obtained:

$$a_1\cdot P_{1,\max}{}^2+b_1\cdot P_{1,\max}+c_1=0 \qquad (1.10)$$

Where:

$$\begin{aligned}&a_1=\left(\frac{2}{M'}\right)^2\cdot(2\cdot k\cdot N'\cdot N''-\left(N''\right)^2-\\&\left(k\cdot N'\right)^2)\end{aligned} \qquad (1.11a)$$

$$b_1 = \left(2 \cdot \frac{M''}{M'}\right)^2 \cdot \left(N' + k \cdot N''\right) + 4 \cdot \left(N' + k \cdot N''\right) \quad (1.11b)$$

$$c_1 = \frac{\left(M''\right)^4}{\left(M'\right)^2} + 2 \cdot \left(M''\right)^2 + \left(M'\right)^2 \quad (1.11c)$$

The coefficient k appears here from the following expression: $Q_1 = k \cdot P_1$.

k depends on power factor ($cos\varphi$) and is calculated: $k = \sqrt{\left(\frac{1}{\cos\varphi}\right)^2 - 1}$.

Power factor is maintained constant.

Quadratic equation (1.10) gives two solutions for $P_{1,\max}$. One solution, which is positive, doesn't make sense because we have consumption in node 1 not generation.

For $P_1 < P_{1,\max}$ the discriminant d of equation (1.6) is positive and we will have an existing steady state. In this case we get two real solutions for U_1'. One of them corresponds to a stable steady state and the other one to an unstable steady state.

For $P_1 = P_{1,\max}$ the discriminant d of equation (1.6) is equal to zero and we are in steady-state stability limit point. In this case we get two identical solutions for U_1'.

For $P_1 > P_{1,\max}$ the discriminant d of equation (1.6) is negative and we will have a non-existing steady state. In this case we get two complex-conjugate solutions for U_1'. These complex solutions in themselves are of no value because such steady states can not exist in reality. But, the fact that the solutions are complex gives us the information that the imposed P_1 is over the power limit $P_{1,\max}$ and that in such conditions steady state can not exist. This criterion can be used when determining $P_{1,\max}$.

In this simple case of power system with only two nodes it is possible to determine $P_{1,\max}$ analytically. In real power systems the analytical determination of power limits is impossible. The power limits in these cases can be determined by successive power flow calculations at each step increasing the consumption with a certain amount in a certain deficit zone. In this case the initial P_1 should be less than $P_{1,\max}$ and in order to perform as less steps as possible it is desirable that the initial P_1 to be as close to $P_{1,\max}$ as possible. The problem is that when choosing the initial P_1, in general, it is difficult to guess the value of P_1 in such a way that $P_1 < P_{1,\max}$. So, from the very beginning we can have a situation when $P_1 > P_{1,\max}$. In this case the conventional power flow programs do not converge to any solution and we have to guess a new value of P_1 less than the precedent one. So, the try-and-error approach is used until the initial value of P_1 satisfies the condition $P_1 < P_{1,\max}$.

If the power flow algorithm permits to get solutions when $P_1 > P_{1,\max}$ we can go towards the power limit point ($P_1 = P_{1,\max}$) from the domain of non-existing steady states. This can improve the convergence of iterative process to the power limit point – $P_{1,\max}$.

Equation (1.2) can also be solved in complex form. First, we will write one more complex equation, which is conjugate of equation (1.2).

$$\overline{U}_1 \cdot \hat{U}_1 + \hat{M} \cdot \overline{U}_1 = \hat{N} \cdot \overline{S}_1 \quad (1.12)$$

From equation (1.12)

$$\hat{U}_1 = \hat{N} \cdot \overline{S}_1 \cdot \frac{1}{\overline{U}_1} - \hat{M} \quad (1.13)$$

Substituting now expression (1.13) in equation (1.2) and performing some transformations the following complex quadratic equation is obtained:

$$\overline{A} \cdot \overline{U}_1^{\,2} + \overline{B} \cdot \overline{U}_1 + \overline{C} = 0 \quad (1.14)$$

Where:

$$\overline{A} = -\hat{M} \quad (1.15a)$$

$$\overline{B} = \hat{N} \cdot \overline{S} - \overline{N} \cdot \hat{S}_1 - \overline{M} \cdot \hat{M} \quad (1.15b)$$

$$\overline{C} = \overline{M} \cdot \hat{N} \cdot \overline{S}_1 \quad (1.15c)$$

The discriminant of quadratic equation (1.14) is a real number.

$$D = \overline{B}^2 - 4 \cdot \overline{A} \cdot \overline{B} = A_1 \cdot P_1^2 + B_1 \cdot P_1 + C_1 \quad (1.16)$$

Where:

$$A_1 = -4 \cdot ((N'')^2 + (k \cdot N')^2 - 2 \cdot k \cdot N' \cdot N'') \quad (1.17a)$$

$$B_1 = 4 \cdot \left[(M')^2 + (M'')^2\right] \cdot (N' + k \cdot N'') \quad (1.17b)$$

$$C_1 = \left[(M')^2 + (M'')^2\right]^2 \quad (1.17c)$$

The power limit $P_{1,\max}$ can be obtained solving the quadratic equation, which is obtained by equating to zero equation (1.16). The positive solution, doesn't make sense because we have consumption in node 1 not generation.

For $P_1 \le P_{1,\max}$ the solutions of quadratic equation (1.14) are also solutions of equation (1.1). $\hat{U}_1$ calculated from (1.13) is equal to conjugate from $\overline{U}_1$.

For $P_1 > P_{1,\max}$ equation (1.1) doesn't have any solutions. Equation (1.14) in this case has solutions but they do not satisfy equation (1.1) if we put instead of $\hat{U}_1$ conjugate from $\overline{U}_1$. When we calculate $\hat{U}_1$ from (1.13) and put this value in equation (1.1) it will be satisfied. $\hat{U}_1$ calculated from (1.13) is not equal to conjugate from $\overline{U}_1$ in this case. From this a criterion can be formulated.

Until the stability limit point $\hat{U}_1$ calculated from (1.13) is equal to conjugate from $\overline{U}_1$ and steady state exists. After the stability limit point $\hat{U}_1$ calculated from (1.13) is not equal to conjugate from $\overline{U}_1$ and in this case steady state can't exist.

The relations between U_1', U_1'' and $\overline{U}_1$, $\hat{U}_1$ are following:

$$\begin{bmatrix} U_1' \\ U_1'' \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 \\ -0.5j & 0.5j \end{bmatrix} \cdot \begin{bmatrix} \overline{U}_1 \\ \hat{U}_1 \end{bmatrix} \quad (1.18)$$

$$\begin{bmatrix} \overline{U}_1 \\ \hat{U}_1 \end{bmatrix} = \begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix} \cdot \begin{bmatrix} U_1' \\ U_1'' \end{bmatrix} \quad (1.19)$$

Until the stability limit point U_1' and U_1'' are real values. After the stability limit point they become complex and this fact tell us that $P_1 > P_{1,\max}$.

Analytical solutions were performed for the following data:

$\overline{Z} = 24.48 + j34.72 \;\; \Omega$,

$\overline{Y} = j208.8 \cdot 10^{-6}$ S, $\overline{U}_2 = 116$ kV,

$P_1 = 0 \ldots 80$ MW (with a step of 0.1 MW),

$\cos\varphi_1 = const = 0.85$ lag.

Analytical solutions for these data are:

Steady-state stability power limit – $P_{1,\max} = 70.31848097515315$ *MW*;

Critical voltage – $U_{1,cr} = 59.39054243505533$ *kV*

Critical angle – $\delta_{21,cr} = 11.58607557117998$ *degree*.

Complex voltage in steady-state stability limit point:

$$\overline{U}_1 = U_1' + jU_1'' = 58.18040597238275 - j11.92798774390883 \text{ kV}.$$

It should be mentioned again that solutions obtained for $P_1 > P_{1,\max}$ do not exist in reality. These fictive solutions can only be of use when determining whether steady state exists or not. The steady-state stability limit point is on the boundary of existing and non-existing steady states. The possibility of getting solutions for non-existing steady states, even they are fictive can help in determination of steady-state stability limit point.

1.2. Numerical solution

The analytical solution is possible only for a system consisting of two nodes. For systems with more than two nodes solutions can be obtained only using a numerical method. Therefore, the investigation of numerical solution is of great importance. Bellow the investigation of numerical solution is performed for the two-node system examined above analytically. Performing complex operations on equation (1.1) and separating it into real and imaginary part the following two real equations are obtained:

$$\begin{cases} G_{11} \cdot \left[(U_1')^2 + (U_1'')^2\right] + U_1' \cdot \left[G_{12} \cdot U_2' - B_{12} \cdot U_2''\right] + U_1'' \cdot \left[G_{12} \cdot U_2'' + B_{12} \cdot U_2'\right] - P_1 = 0 \\ B_{11} \cdot \left[(U_1')^2 + (U_1'')^2\right] + U_1' \cdot \left[B_{12} \cdot U_2' + G_{12} \cdot U_2''\right] + U_1'' \cdot \left[-G_{12} \cdot U_2' + B_{12} \cdot U_2''\right] + Q_1 = 0 \end{cases} \quad (1.20)$$

Where: $G_{11} = \mathrm{Re}(\overline{Y}_{11})$, $B_{11} = \mathrm{Im}(\overline{Y}_{11})$, $G_{12} = \mathrm{Re}(\overline{Y}_{12})$, $B_{12} = \mathrm{Im}(\overline{Y}_{12})$.

In case when bus 1 is a PV-bus the following two equations are written:

$$\begin{cases} G_{11}\cdot\left[(U_1')^2 + (U_1'')^2\right] + \\ U_1'\cdot\left[G_{12}\cdot U_2' - B_{12}\cdot U_2''\right] + \\ U_1''\cdot\left[G_{12}\cdot U_2'' + B_{12}\cdot U_2'\right] - P_1 = 0 \\ (U_1')^2 + (U_1'')^2 - U_1^2 = 0 \end{cases} \quad (1.21)$$

The variables of equations (1.20) are U_1' and U_1'', which are respectively real and imaginary part of $\overline{U}_1$. As was shown above these are real values in case when steady state exists and become complex in case of non-existing steady states. If the iterative process is looking for solutions only from the domain of real values the convergence is impossible for the cases when $P_1 > P_{1,\max}$. To get solutions for non-existing steady states we have to extend the domain of solutions to the domain of complex numbers. This can be done by assigning at the beginning of iterative process complex values to U_1' and/or U_1''. For example: $U_1' = (110 + 1j)$ and $U_1'' = 0$. So, now we consider U_1' and U_1'' as complex variables. In this case the iterative process is looking for solutions from the domain of complex values.

When $P_1 \le P_{1,\max}$ the iterative process converges to only real solutions; complex solutions are not possible in this case.

When $P_1 > P_{1,\max}$ the iterative process converges to only complex solutions; real solutions do not exist in this case.

The magnitude of the Jacobian determinant in steady-state stability limit point calculated using analytical solutions presented above is equal to $9.866369354827045\cdot 10^{-16}$. In fact, it should be zero in this point, but because of roundoff computer errors it is not ideally equal to zero. The Jacobian is singular in this case. So, if we assume that when calculated numerically solutions can be obtained absolutely precise, we would have convergence problems in steady-state stability limit point.

In practice, numerical solutions are obtained with a certain precision. Solutions obtained numerically slightly differ from the analytical solutions. The magnitude of the Jacobian determinant in steady-state stability limit point calculated for solutions obtained numerically with a precision of 0.001 kV is equal to $2.13412275027657\cdot 10^{-4}$. As we can see, the difference between the Jacobian calculated numerically and analytically is considerable. The Jacobian is not singular when it is calculated using numerical solutions, which are approximate. Therefore the iterative process converges in steady-state stability limit point.

A series of power flow calculations were performed for $P_1 = 0\ldots 80$ MW (with a step of 0.1 MW). At every step voltages are calculated with a precision of 0.001 kV. The magnitude of Jacobian determinant is decreasing to a certain minimum value and after that it is increasing. The minimum value of magnitude of Jacobian determinant (0.1233) corresponds to $P_1 = 70.3$ MW, which can be considered as $P_{1,\max}$. Comparing this value (70.3) with $P_{1,\max}$ calculated analytically (70.31848097515315) we can conclude that the precision is quite satisfactory. Since the approximate solutions are used when calculating Jacobian its determinant doesn't decrease to zero (the Jacobian doesn't become singular) and we do not have convergence problems whatever P_1 is.

The convergence has been studied for two cases. In first case for every P_1 flat start was used. The convergence in this case can be considered satisfactory even when $P_1 \ge P_{1,\max}$ - 14 iterations and less are necessary to get solutions. In second case for $P_1 = 0$ flat start was used and for other P_1 the solutions from the previous step were used. When using non-flat start the convergence is better than in case of flat start. The solutions are obtained in maximum 5 iterations.

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