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The method of calculating LC parameters of balancing compensators in a three-phase four-wire circuit for an unbalanced linear receiver

The method of calculating LC parameters of balancing compensators in a three-phase four-wire circuit for an unbalanced linear receiver

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Abstract. The article uses the Currents' Physical Component (CPC) power theory to determine the parameters of the reactive compensator in the case of a three-phase four-wires unbalanced receiver on a non-sinusoidal periodic power supply. The basic relationships and the way of calculating the compensator reactance is presented. The compensator was built from two parts: the first working in the star-topology, and the second in the triangle-topology. The correctness of the formulas has been verified by presenting the results of calculations for an exemplary system.

1. Introduction

This article presents the method of power compensation using a reactive compensator in a three-phase four-wires circuit for periodic, non-sinusoidal waveforms using the Currents' Physical Component (CPC) power theory. A linear, three-phase, four-wire and unbalanced receiver, with three-phase source with non-sinusoidal waveforms was adopted for the analysis. The general idea of the CPC theory was taken from several articles [1-5,16]. An analysis on a non-linear receiver is discussed in [6-15]. Decomposition of the current components in a four-wire circuit was developed in [1,2,5]. The power compensation in the four-wire circuit in the case of sinusoidal waveforms is shown in [5]. In this article, an extension of this material will be presented taking into account the possibility of power compensation in a four-wire system with non-sinusoidal periodic forcing.

2. Theoretical foundations and dependencies

The CPC power theory was derived by Professor Czarnecki. Currently, three-phase four-wire circuits are considered in this theory. For such circuits, power compensation was developed in [2]. Theoretical equations and the reactive power compensation algorithm was developed there using a two-part reactive compensator. This article will show the practical use of these formulas.

The power factor in a three-phase four-wire system with harmonic waveforms can be expressed as:

$$\lambda = \frac{P}{S} = \frac{P}{\sqrt{P^2 + Q^2 + D_\mathrm{s}^2 + D_\mathrm{u}^{\mathrm{p}2} + D_\mathrm{u}^{\mathrm{n}2} + D_\mathrm{u}^{\mathrm{z}2}}} = \frac{\|\boldsymbol{i}_\mathrm{a}\|}{\|\boldsymbol{i}\|} = \frac{\|\boldsymbol{i}_\mathrm{a}\|}{\sqrt{\|\boldsymbol{i}_\mathrm{a}\|^2 + \|\boldsymbol{i}_\mathrm{s}\|^2 + \|\boldsymbol{i}_\mathrm{r}\|^2 + \|\boldsymbol{i}_\mathrm{u}^\mathrm{p}\|^2 + \|\boldsymbol{i}_\mathrm{u}^\mathrm{n}\|^2 + \|\boldsymbol{i}_\mathrm{u}^\mathrm{z}\|^2}}\,. \tag{1}$$

This means that the reduction of the power factor is influenced not only by the Q reactive power, but also by the D_s scattered power and the unbalanced powers: D_u^p, D_u^n, D_u^z. The power factor can also be expressed in the field of three-phase rms current values. Only the $\boldsymbol{i}_a$ active component is the usable part of $\boldsymbol{i}$ current. In order to improve the power factor, the values of the other components should be reduced.

A reactive compensator connected in parallel to a three-phase four-wire receiver (figure 1) is able to influence the following components: reactive and unbalanced current. The scattered current can only be compensated in a serial configuration in each phase separately, thus changing the receiver's operating point. For a parallel compensator, the minimum value of the power factor will reach the value of:

$$\lambda = \frac{\|\boldsymbol{i}_a\|}{\sqrt{\|\boldsymbol{i}_a\|^2 + \|\boldsymbol{i}_s\|^2}}. \tag{2}$$

To improve the power factor, a compensator should be built, which consists of two elements: the first in the star topology and the second in the triangle topology. This method was discussed in [2,5].

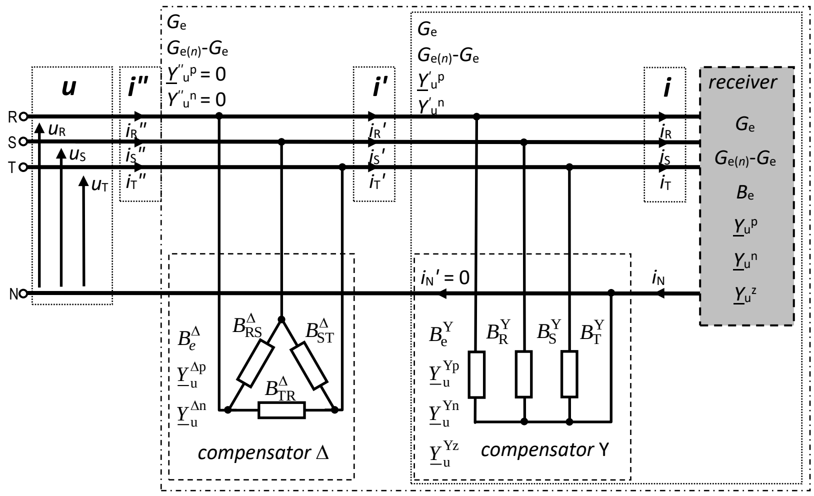

Figure 1. The Δ and Y compensators connected to the receiver.

The first segment of this compensator is connected in the Y topology and it affects the reactive current $\boldsymbol{i}_r$ and the unbalanced current of the zero sequence $\boldsymbol{i}_u^z$. The susceptances of this segment B_R^Y, B_S^Y and B_T^Y are equals:

$$\begin{cases} B^{Y}_{R(n)} = -2\Im m\left\{\underline{Y}^{z}_{u(n)}\right\} - B_{e(n)} \\ B^{Y}_{S(n)} = -s\sqrt{3}\Re e\left\{\underline{Y}^{z}_{u(n)}\right\} + \Im m\left\{\underline{Y}^{z}_{u(n)}\right\} - B_{e(n)} \\ B^{Y}_{T(n)} = s\sqrt{3}\Re e\left\{\underline{Y}^{z}_{u(n)}\right\} + \Im m\left\{\underline{Y}^{z}_{u(n)}\right\} - B_{e(n)}. \end{cases} \quad \text{where: } s = \begin{cases} 1 & \mathrm{n} = 3k+1 \quad \text{(positive sequence)} \\ -1 & \mathrm{n} = 3k+2 \quad \text{(negative sequence)} \end{cases} \tag{3}$$

To determine the parameters of the compensator Δ, an analysis presented in [2] regarding the three-wire system can be used. However, for this purpose, the resultant physical quantities describing the receiver together with the Y compensator should be determined. These quantities are marked with a single "*prim*" symbol.

$$\underline{Y}'_{RS(n)} = \frac{\left(\underline{Y}_{R(n)} + jB^{Y}_{R(n)}\right)\cdot\left(\underline{Y}_{S(n)} + jB^{Y}_{S(n)}\right)}{3\cdot G_{e(n)}},$$
$$\underline{Y}'_{ST(n)} = \frac{\left(\underline{Y}_{S(n)} + jB^{Y}_{S(n)}\right)\cdot\left(\underline{Y}_{T(n)} + jB^{Y}_{T(n)}\right)}{3\cdot G_{e(n)}}, \tag{4}$$

$$\underline{Y}'_{\mathrm{TR}(n)} = \frac{\left(\underline{Y}_{\mathrm{T}(n)} + jB^{\mathrm{Y}}_{\mathrm{T}(n)}\right)\cdot\left(\underline{Y}_{\mathrm{R}(n)} + jB^{\mathrm{Y}}_{\mathrm{R}(n)}\right)}{3\cdot G_{\mathrm{e}(n)}}, \qquad \text{for } n \neq 3k+3.$$

Parallel connection of the receiver and the Y compensator is a equivalent circuit. Parameters of the equivalent circuit are presented after transposition into the triangle topology. The equivalent circuit is described by the following quantities, specific to the Δ topology:

- the equivalent admittance, equals:

$$\underline{Y}'_{\mathrm{e}(n)} = \underline{Y}'_{\mathrm{RS}(n)} + \underline{Y}'_{\mathrm{ST}(n)} + \underline{Y}'_{\mathrm{TR}(n)} = G'_{\mathrm{e}(n)}, \tag{5}$$

- the unbalanced admittance, equals:

$$\underline{Y}'_{\mathrm{u}(n)} = -\left(\underline{Y}'_{\mathrm{ST}(n)} + \beta^{*}_{(n)}\underline{Y}'_{\mathrm{TR}(n)} + \beta_{(n)}\underline{Y}'_{\mathrm{RS}(n)}\right). \tag{6}$$

$$\begin{cases} B^{\Delta}_{\mathrm{RS}(n)} = \frac{1}{3}\left(s\sqrt{3}\Re e\{\underline{Y}'_{\mathrm{u}(n)}\} - \Im m\{\underline{Y}'_{\mathrm{u}(n)}\}\right) \\ B^{\Delta}_{\mathrm{ST}(n)} = \frac{2}{3}\Im m\{\underline{Y}'_{\mathrm{u}(n)}\} \\ B^{\Delta}_{\mathrm{TR}(n)} = \frac{1}{3}\left(-s\sqrt{3}\Re e\{\underline{Y}'_{\mathrm{u}(n)}\} - \Im m\{\underline{Y}'_{\mathrm{u}(n)}\}\right), \end{cases} \quad \text{where: } s = \begin{cases} 1 & \mathrm{n} = 3k+1 \quad \text{(positive sequence)} \\ -1 & \mathrm{n} = 3k+2 \quad \text{(negative sequence)} \end{cases} \tag{7}$$

The compensator Δ with susceptances determined in accordance with (7) causes zeroing of the current components of the source: reactive $\boldsymbol{i}_{\mathrm{r}(n)}$ and unbalanced $\boldsymbol{i}_{\mathrm{u}(n)}$ current. After a two-segment compensation, the power factor of the system takes the minimum value and it is equal to (2).

3. Calculation example

The single-phase receiver was built from a serial RL connection and was powered from a three-phase source with symmetrical voltage of the following waveform: $u_{\mathrm{R}} = \sqrt{2}\cdot\Re e\{230e^{j\omega_1 t} + 10e^{j5\omega_1 t}\}$ V, $\omega_1 = 2\pi 50$ rad/s.

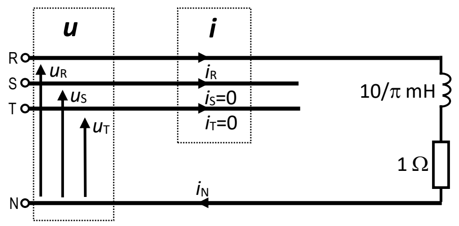

Figure 2. One-phase receiver connected to a three-phase, non-sinusoidal, four-wire source.

This receiver can be described as follows:

n	$\underline{Y}_{\mathrm{R(n)}}$ [S]	$\underline{Y}_{\mathrm{e(n)}}$ [S]	$\underline{Y}^{\mathrm{z}}_{\mathrm{u}(n)}$ [S]	$\underline{Y}^{\mathrm{p}}_{\mathrm{u}(n)}$ [S]	$\underline{Y}^{\mathrm{n}}_{\mathrm{u}(n)}$ [S]
1	$1/2 - j\,1/2$	$1/6 - j\,1/6$	$1/6 - j\,1/6$	0	$1/6 - j\,1/6$
5	$1/26 - j\,5/26$	$1/78 - j\,5/78$	$1/78 - j\,5/78$	$1/78 - j\,5/78$	0

where:

the equivalent admittances were determined from $\underline{Y}_{\mathrm{e}(n)} = 1/3\left(\underline{Y}_{\mathrm{R}(n)} + \underline{Y}_{\mathrm{S}(n)} + \underline{Y}_{\mathrm{T}(n)}\right)$,

the unbalanced admittances were determined from [2 eq.(23-25)].

Active power is taken only in one phase and it is equal to $P = P_{\mathrm{R}} = \sum_n \Re e\{\underline{Y}_{\mathrm{R}(n)}\}\cdot U^2_{\mathrm{R}(n)} = 26454$ W.

For a balanced power source, the effective value of three-phase voltage is $\|\boldsymbol{u}\| = \sqrt{\sum_n 3 \cdot U_{\mathrm{R}(n)}^2} = 398.75\,\mathrm{V}$.

This means that the equivalent conductance is equal to $G_\mathrm{e} = P / \|\boldsymbol{u}\|^2 = 166.38\,\mathrm{mS}$.

The rms values of the individual three-phase current components are equal to:

$\|\boldsymbol{i}_\mathrm{a}\| = G_\mathrm{e}\|\boldsymbol{u}\| = 66.34\,\mathrm{A}$, $\|\boldsymbol{i}_\mathrm{s}\| = \sqrt{3\sum_n \left(G_{\mathrm{e}(n)} - G_\mathrm{e}\right)^2 \cdot U_{\mathrm{R}(n)}^2} = 2.66\,\mathrm{A}$, $\|\boldsymbol{i}_\mathrm{r}\| = \sqrt{3\sum_n B_{\mathrm{e}(n)}^2 \cdot U_{\mathrm{R}(n)}^2} = 66.41\,\mathrm{A}$,

$\|\boldsymbol{i}_\mathrm{u}^\mathrm{p}\| = \sqrt{3\sum_n \left(Y_{\mathrm{u}(n)}^\mathrm{p}\right)^2 \cdot U_{\mathrm{R}(n)}^2} = 1.13\,\mathrm{A}$, $\|\boldsymbol{i}_\mathrm{u}^\mathrm{n}\| = \sqrt{3\sum_n \left(Y_{\mathrm{u}(n)}^\mathrm{n}\right)^2 \cdot U_{\mathrm{R}(n)}^2} = 93.90\,\mathrm{A}$, $\|\boldsymbol{i}_\mathrm{u}^\mathrm{z}\| = \sqrt{3\sum_n \left(Y_{\mathrm{u}(n)}^\mathrm{z}\right)^2 \cdot U_{\mathrm{R}(n)}^2} = 93.90\,\mathrm{A}$.

Thus, the rms value of the three-phase current is equal to:

$$\|\boldsymbol{i}\| = \sqrt{\|\boldsymbol{i}_\mathrm{a}\|^2 + \|\boldsymbol{i}_\mathrm{s}\|^2 + \|\boldsymbol{i}_\mathrm{r}\|^2 + \|\boldsymbol{i}_\mathrm{u}^\mathrm{p}\|^2 + \|\boldsymbol{i}_\mathrm{u}^\mathrm{n}\|^2 + \|\boldsymbol{i}_\mathrm{u}^\mathrm{z}\|^2} = 162.65\,\mathrm{A}.$$

This means that the power factor of this system is equal to:

$$\lambda = \frac{\|\boldsymbol{i}_\mathrm{a}\|}{\sqrt{\|\boldsymbol{i}_\mathrm{a}\|^2 + \|\boldsymbol{i}_\mathrm{s}\|^2 + \|\boldsymbol{i}_\mathrm{r}\|^2 + \|\boldsymbol{i}_\mathrm{u}^\mathrm{p}\|^2 + \|\boldsymbol{i}_\mathrm{u}^\mathrm{n}\|^2 + \|\boldsymbol{i}_\mathrm{u}^\mathrm{z}\|^2}} = 0.408\,.$$

In order to improve the power factor, the compensator parameters should be selected. The first part, i.e. the compensator working in the Y topology, is made of susceptance, whose values are determined from (3):

n	$B_{\mathrm{R}(n)}^\mathrm{Y}$ [S]	$B_{\mathrm{S}(n)}^\mathrm{Y}$ [S]	$B_{\mathrm{T}(n)}^\mathrm{Y}$ [S]
1	$1/2$	$-\sqrt{3}/6$	$\sqrt{3}/6$
5	$5/26$	$\sqrt{3}/78$	$-\sqrt{3}/78$

There are infinitely many possibilities to implement such a compensator. Physical implementation requires a different terminal structure in each phase.

The physical implementation of these admittances are reactance terminals whose structures and parameters are:

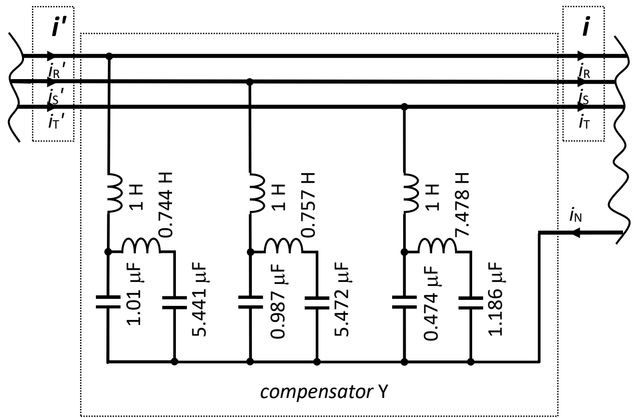

Figure 3. Physical realization of the Y compensator, based on phase susceptances.

The next step is to make a transfiguration of the system from the topology of the star, built from parallel connection of the receiver and compensator Y to the topology of the triangle. The parameters of the equivalent circuit, is determined from (4), hence:

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n	$\underline{Y}'_{RS(n)}$ [S]	$\underline{Y}'_{ST(n)}$ [S]	$\underline{Y}'_{TR(n)}$ [S]	$\underline{Y}'_{e(n)}$ [S]	$\underline{Y}'_{u(n)}$ [S]
1	$-j\sqrt{3}/6$	$1/6$	$j\sqrt{3}/6$	$1/6$	$1/3$
5	$j\sqrt{3}/78$	$1/78$	$-j\sqrt{3}/78$	$1/78$	$1/39$

where:

the equivalent admittances has determined from (5),
the unbalanced admittances has determined from (6),

The susceptances of the Δ compensator are determined from (7):

n	$B^{\Delta}_{RS(n)}$ [S]	$B^{\Delta}_{ST(n)}$ [S]	$B^{\Delta}_{TR(n)}$ [S]
1	$\sqrt{3}/9$	0	$-\sqrt{3}/9$
5	$-\sqrt{3}/117$	0	$\sqrt{3}/117$

The implementation of the susceptances is the LC ladder system, whose parameters have been determined analogically to the previous segment of the compensator. The following is one of the possible implementations of the system.

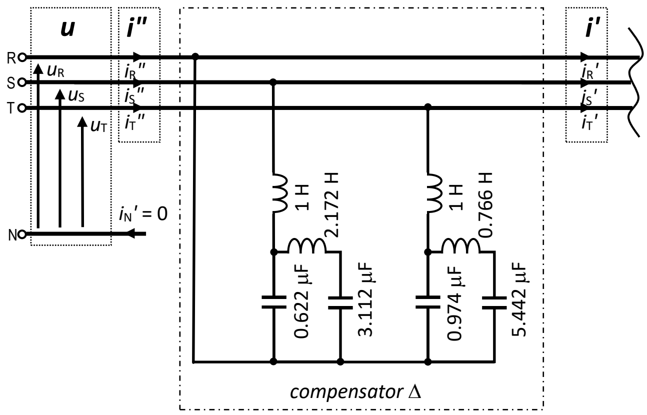

Figure 4. Physical realization of the Δ compensator, based on phase susceptances.

The two-fold Y and Δ compensation resets the reactive current $\boldsymbol{i}_r$ and the unbalanced current $\boldsymbol{i}_u$. The scattered current $\boldsymbol{i}_s$ still remains, which causes the power factor not to equal to one. From (2) the following value is obtained:

$$\lambda = \frac{\|\boldsymbol{i}_a\|}{\sqrt{\|\boldsymbol{i}_a\|^2 + \|\boldsymbol{i}_s\|^2}} = \frac{66.3423}{\sqrt{66.3423^2 + 2.6622^2}} = 0.9992\,.$$

Any further improvement in the power factor consists in reducing the influence of the scattered current $\boldsymbol{i}_s$. This is only possible by performing serial compensation. The series compensator leads to complete compensation only when the real part of the resultant impedance of the receiver along with the Y and Δ compensators does not depend on the frequency. In addition to this, the construction of the third compensator segment, working in series in each phase is economically unprofitable.

4. Conclusions

The reactive compensation in three-phase four-wire systems is more complex than compensation in three-wire systems. Two segments of the reactive compensator are required to reduce the power factor. The paper presents the basics of reactive compensation of linear loads with constant parameters. In fact, these parameters change over time. In this situation, an adaptive compensator may be needed. However, the discussion about such adaptive compensation goes beyond the scope of this article.

A source with zero internal impedance was considered. It is a simplification that may be unacceptable in real systems. Compensation with real source remains a problem which must be considered in future articles.

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