EVALUATION OF THE OPTIMAL SOLAR FRACTION FOR A DISTRICT HEATING SYSTEM

EVALUAREA FRACŢIEI SOLARE OPTIME PENTRU UN SISTEM DE TERMOFICARE

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Abstract: *This paper addresses the issue of promoting solar thermal energy in district heating systems and determining the optimal solar fraction in the system heat generation. Despite the bad soviet past of this technology, it is time to insist on promoting a new generation of district heating systems (fourth one), free from the shortcomings of the past, as pillars of the future decarbonized energy systems, based on the use of renewable energy sources and heat recyclable waste. The problem of establishing the solar fraction, integrated in a district heating system is an optimization problem. Even if solar radiation is free and available virtually everywhere - solar technology and infrastructure (solar collectors, heat reservoirs, underground heat accumulators, pumps for circulating the working agent, heat pumps for discharge, etc.) are quite expensive. Based on the application of the criterion of minimum total expenditures, a mathematical techno-economic model was developed, which allowed to obtain an analytical expression for the calculation of the solar optimal fraction, on the one hand, and the share of hot water boilers on natural gas or biomass - on the other hand. This assessment was made on both the power and energy dimensions. The numerical calculations performed demonstrate the application of this analytical tool in the pre-feasibility studies of solar district heating systems.*

Keywords: district heating system, solar heat plants, solar thermal fraction, heat tanks, underground thermal energy storages, heat pumps, discounted total cost.

Rezumat: *În această lucrare este abordată problema promovării energiei termice solare în sistemele de termoficare și determinării fracției optime solare în structura surselor de căldură integrate în sistem. În ciuda unui*

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trecut sovietic nefast al acestei tehnologii, e cazul de a promova cu insistență o nouă generație de sisteme de termoficare (generația a patra), lipsite de neajunsurile din trecut, ca piloni ai viitorului sistem energetic decarbonizat și bazat pe utilizarea surselor regenerabile de energie și deșeurilor reciclabile de căldură. Problema stabilirii fracției solare, integrate într-un sistem de termoficare este o problemă complexă de optimizare. Chiar dacă radiația solară este gratuită și disponibilă practic peste tot – tehnologia și infrastructura solară (colectoarele solare, rezervoarele de căldură, acumulatoarele subterane de căldură, pompele pentru circulația agentului de lucru, pompele de căldură pentru descărcare etc.) sunt suficient de costisitoare. În baza aplicării criteriului cheltuielilor totale minime a fost dezvoltat un model matematic tehnico-economic, care a permis de a obține o expresie analitică de calcul al fracției optime solare, pe de-o parte, și cotei cazanelor de apă fierbinte pe gaze naturale sau biomasă – pe de altă parte. Această evaluare a fost realizată atât pe dimensiunea putere, cât și energiei. Calculele numerice realizate demonstrează utilitatea aplicării acestui instrument analitic în studiile de prefezabilitate a sistemelor solare de termoficare.

Cuvinte cheie: sistem de termoficare, centrale termice solare, fracția solară termică, rezervoare de căldură, acumulatoare subterane de energie termică, pompe de căldură, cheltuielile totale actualizate.

1. Introduction

Climate change today represents the greatest challenge for the future of humanity. The uncontrolled burning of fossil fuels over a century has led to a continuous increase in the concentration of *greenhouse gases* (GHG) in the atmosphere, which is an obstacle in the process of dissipating the earth's heat into outer space, and as a result, leads to an increase in the temperature at the surface of the planet - with adverse effects on the climate on Earth.

In accordance with the provisions of the Paris Agreement and based on the concept of *climate neutrality* (*net-zero GHG emissions),* global warming until 2050 must be kept below at most 1.5 ^{0}C compared to pre-industrial times. In order to achieve this goal, the countries of the world in the following decades must considerably reduce GHG emissions, mainly by:

(i) *reducing energy consumption* in society - by promoting energy efficiency measures along the entire "production-consumption" chain, as well as through

(ii) *the transition to the use of 100% renewable energy*.

The sun is mankind's most important source of energy, followed by wind energy and bioenergy, derived from solar energy, and other sources.

Thus, the energy future of the planet will rely mainly on solar energy (thermal and photovoltaic), converted into many other forms of final energy.

District heating systems (or district heating systems, DHS) are known for their ability to integrate renewable sources into the consumer supply system and respectively to reduce the associated GHG emissions [1].

Heating systems have a very important role to play in the supply of heat and cold within the new decarbonized energy systems. Unfortunately, in the past this technology was strongly discredited and to this day it is regarded in society with little confidence. In the period 1980-2000 in the Republic of Moldova there were about 50 systems of this kind, and currently there are only two - in the municipalities of Chisinau and Balti.

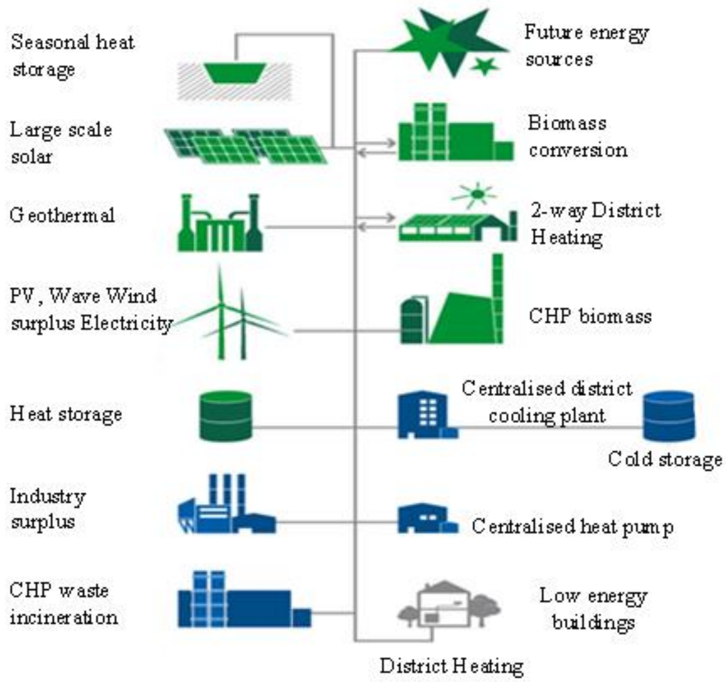

Figure 1. The constituent elements of a modern DHS, 4th generation [2]

Currently, heating, all over the world, is oriented towards modern systems, known as the 4th generation (DHS-G4), fig. 1, which provide for supplying consumers with heat obtained from renewable sources, such as solar thermal power plants, biomass, geothermal power plants, heat pumps, waste heat sources from industrial sectors. DHS-G4, as a rule, are equipped with a short-term and long-term heat storage infrastructure, allow the utilization of surplus electricity produced by variable renewable sources and others.

2. Integrating solar power plants into heating systems – an indispensable priority

In the last two decades several European states (Germany, Austria, France, Denmark, Holland, Spain) and around the world (China, USA, India, Turkey, South Korea, Brazil) have made a great effort in investigating and implementing systems heat production solar, integrated in DHS ([3] - [7]).

Towards the end of 2020, solar heating and cooling systems worldwide had a total capacity of 501 GW_t (fig. 2), bringing an annual thermal energy contribution of approx. 407 TWh, equivalent to the saving of 43.8 million tons of oil and the reduction of GHG emissions by approx. 141 million tons. In the field of solar energy use - heating, compared to solar energy applications, benefited the most. Globally in 2020 there were 262 solar thermal power plants in operation, integrated in DHS, with a total installed capacity of 1410 MW_t (2.01 million m^2 solar collectors) [8].

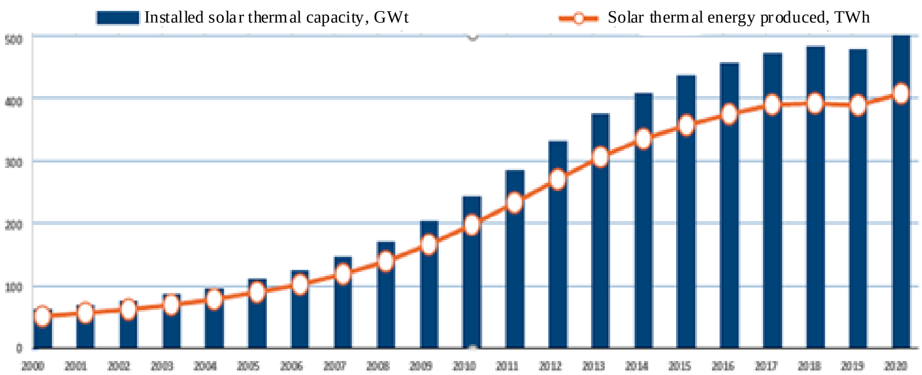

Figure 2. Installed capacity and energy produced annually globally by solar thermal power plants in the period 2000-2020 [8]

On this segment of the global thermal energy market, Denmark has been the absolute leader for about a decade - both in terms of the number of DHS, with solar thermal sources, and the area of installed solar collectors. This success was due to the energy policy favorable to the field, promoted by Denmark over the years, and in 2020 when national priorities changed (in favor of heat pumps) - Germany has already become the leader in promoting solar thermal plants in DHS.

In Europe, as a finding, 14 solar thermal systems (> 500 m^2) were implemented in 2020, integrated in DHS, of which:

- 7 were built in Germany (31200 m^2),

- 4 in Denmark (14,600 m²),
- 2 in Austria (6 571 m²) and
- 1 in Switzerland (784 m²).

Thus, by 2021, the EU member states, which had an important solar contribution in DHS, were:

- Denmark (in total - 124 solar heating systems, with more than 1.5 million square meters of collecting surface and more than 1000 MW of thermal capacity),
- Germany (43), Austria (19), Sweden (22) and Poland (8). For comparison - China (18 systems).

Figure 3. The heating system in Marstal, Denmark - a modern and efficient system [9], [10]

Figure 3 shows an example of a performing solar system, which includes:

- solar panels: field #1 - 8000 m^2, field #2 - 9300 m^2, field #3 - 15 000 m^2;
- ORC cogeneration plant - 750 kW$_e$, thermal oil boiler 4.0 MW;
- seasonal accumulator #1 - 10,000 m^3, seasonal accumulator #2 - 75,000 m^3;
- metal storage accumulator - 2,000 m^3;
- heat pump of 1.5 MW$_t$.

One of the most widespread models of flat thermal solar collectors, used all over the world in solar heating systems, are of the Arcon-Sunmark type (originating in Denmark). Recently (2020) this trademark was taken over by the Austrian giant GREENoneTEC, with the relocation of the means of production, and starting from 2021 the production and distribution of this type of solar thermal collectors is carried out in Austria [6].

Large surface collectors, intended for heating and industrial applications, are manufactured in standard sizes of 5, 8, 10 and 13 m² - with single or double glazing, with anti-reflective coated glass on both sides (fig. 4). These collectors allow installation with a crane.

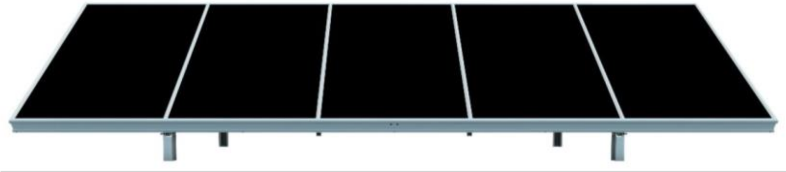

Figure 4. Arcon - Sunmark flat thermal solar collector GK HT 13.6 , with five panels - for large solar systems[6]

In new times, when during a single year the price of natural gas in the country has increased four times, the priority is the integration of renewable sources into the heating systems and, first of all, solar thermal power plants and solid biomass power plants. The schematic structure of a modern DHS, based on the use of renewable energy sources, is shown in fig. 5.

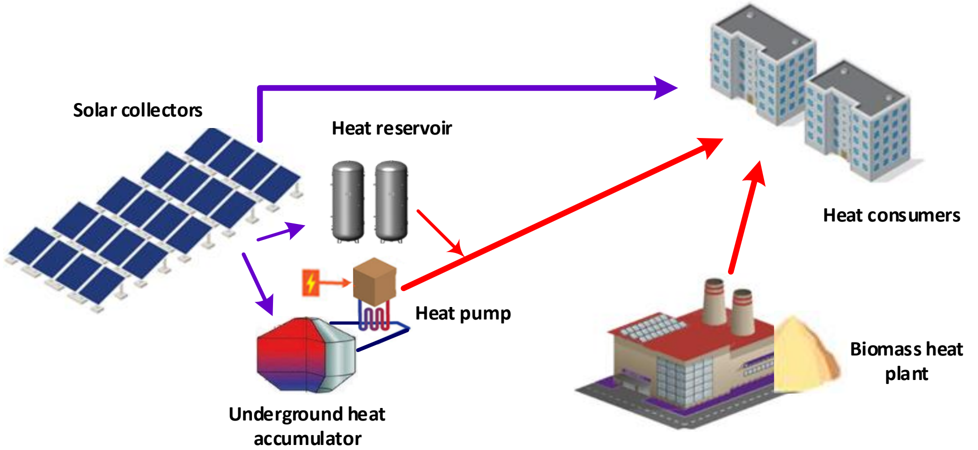

Figure 5. The component elements of a modern heating system

[6] https://www.greenonetec.com/en/units/large-scale-projects/arcon-sunmark/

3. Problem formulation

We will consider a heating system (DHS), which includes two heat sources - a solar source (SS, solar thermal collector system), put in competition with another source, called reference source (RS); as RS in this work are considered: (i) hot water boilers, fed with natural gas and (ii) solid biomass boilers.

It is of interest to determine the optimal share of the solar source in the heat generation structure in DHS.

The solar fraction/share is defined as the ratio of the contribution of the solar source to cover the heat consumption, to the total amount of heat supplied in DHS. This can be tracked both for a moment of time (hour) - that is characteristic of the annual maximum load, and for a period of time - of a year. In the first case, we will talk about the solar thermal share in covering the annual maximum load, denoted by (*x*), and in the second - the solar thermal share in covering the annual heat consumption in the considered system (*X*).

Based on the above for the two quantities { $\boldsymbol{x^{solar}}$ and $\boldsymbol{X^{solar}}$} we can write:

$$\boldsymbol{x^{solar}} = \boldsymbol{q_{nom}^{solar}} / \boldsymbol{q_{M}^{SACET}} \quad \text{and} \tag{1}$$

$$\boldsymbol{X^{solar}} = \boldsymbol{Q_{an}^{solar}} / \boldsymbol{Q_{an}^{SACET}}. \tag{2}$$

In this study we will assume that the annual maximum heat load as well as the annual heat consumption in DHS does not vary from year to year. The duration of the study period *T* is accepted at the level of the lifetime of the heat sources (T = 20 years). A distinctive aspect of the given approach is the consideration that a number of problem parameters (annual fuel, O&M, etc.) evolve over the study period. The annual values of the evolutionary parameters (z_t) are estimated using exponential functions of the form: $z_t = z_{t_0} \cdot (1 + r_z)^{t-t_0}$, where z_{t_0} represents the known value of the parameter $\boldsymbol{z}$ in the reference year t_0, and r_z - the rate of annual increase/decrease of its value.

The modeling of the volume of the annual heat production, carried out by the sources integrated in DHS (Cs + Cz), is based on two key elements: the analytical description of the annual ranked heat load curve (CC-ST), with the use of a power-function and the analytical calculation of the area of the surfaces resulting from dividing the curve classified into: the base area and the peak area - for any value of the fraction $\boldsymbol{x}$ considered in the optimization process.

The problem of determining the optimal solar fraction in the structure of the heat generated in a heating system is an optimization problem to be solved based on the economic criterion. *The objective function* of the problem is represented by the function of the minimum updated total expenses, related to the construction and subsequent operation of solar thermal power plants and thermal power plants equipped with hot water, biomass or natural gas boilers. All components of the objective function are to be expressed by the variable *x* .

From the condition - $\partial\mathbf{F}/\partial\mathbf{x} = \mathbf{0}$, for $\partial^2\mathbf{F}/\partial\mathbf{x}^2 \geq \mathbf{0}$, the optimal value will result $\mathbf{x}_{opt}$.

4. Mathematical modeling of the problem and analytical solution

The mathematical model of the problem, which naturally corresponds to a dynamic model, is finally presented by means of a static model - equivalent to the dynamic one [11-12]. The static-equivalent model, compared to the dynamic model, in addition to representing a compact description of the evolution of annual expenses over time and considerably simplifying the calculation of CTA, allows to operate with analytical expressions, on which equivalent transformations can easily be made [11].

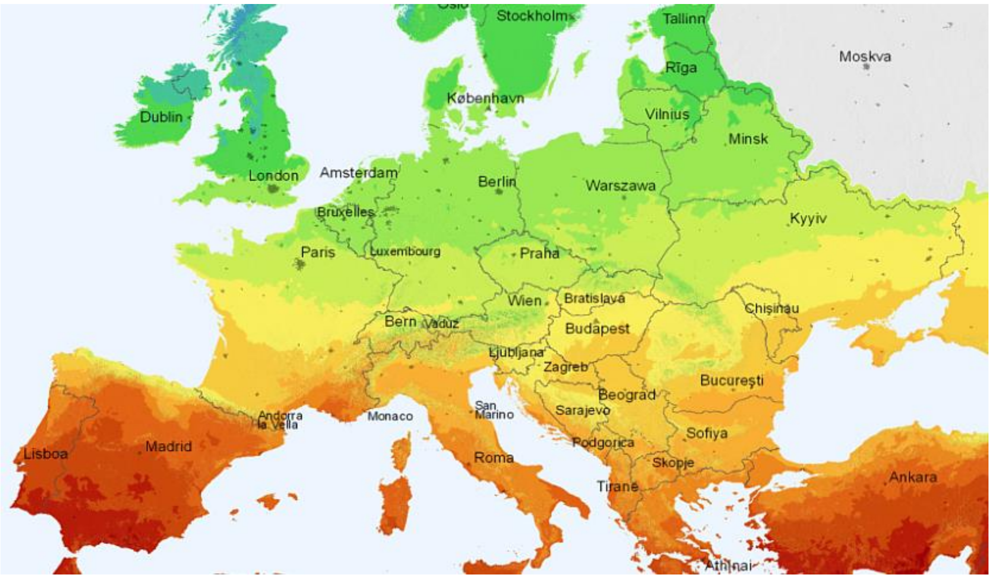

Figure 6. Distribution of solar radiation intensity in Europe
Source: Wikipedia

Investment expenses

For a share *x* of the solar collectors (Cs), considered in the study, the installed power of this source is:

$$q_{nom}^{Cs} = x \cdot q_M^{sarc}, \tag{3}$$

and the power of the boilers (Cz) turns out to be –

$$q_{nom}^{Cz} = (1 - x) \cdot q_M^{sarc}. \tag{4}$$

It should be noted that in these calculations the notion of "solar collectors" essentially reflects the so-called "solar block", which includes solar collectors, heat accumulators and heat pumps. This means that the expenses related to "solar collectors" reflect all the expenses (with investment, O&M) of the entire solar block.

The value of the investment in an energy source is often expressed by the specific investment and the installed power. In the hypothesis of building the energy sources in a period that does not exceed one year, for the total expenses with the investment in the two sources (Cs + Cz) we can write:

$$CTA_I^{Cs} = I^{Cs} = q_{nom}^{Cs} \cdot i_s^{Cs} = q_M^{sarc} \cdot \boldsymbol{x} \cdot i_s^{Cs}, \tag{5}$$

$$CTA_I^{Cz} = I^{Cz} = q_{nom}^{Cz} \cdot i_s^{Cz} = q_M^{sarc} \cdot (\boldsymbol{1} - \boldsymbol{x}) \cdot i_s^{Cz}. \tag{6}$$

where q_{nom}^{Cs} and q_{nom}^{Cz} represent the nominal powers of the sources, and i_s^{Cs}- i_s^{Cz} the related specific investments.

Thus, for the total expenses with the investment in Cs and Cz, which covers the thermal load of DHS, it results:

$$CTA_I^{Cs+Cz} = q_M^{sarc} \cdot [x \cdot (i_s^{Cs} - i_s^{Cz}) + i_s^{Cz}]. \tag{7}$$

We will note: $A_I = i_s^{Cs} - i_s^{Cz}$, (8)

so that, finally, for the total expenses with the investment in the two sources we can write:

$$\boldsymbol{CTA_I^{Cs+Cz} = q_M^{sarc} \cdot [x \cdot A_I + i_s^{Cz}].} \tag{9}$$

O&M expenses

The annual O&M expenses $C_{O\&M,t}$ are often determined by applying a quota $k_{O\&M}$ to the total investment value *I*, related to the respective heat source:

$$C_{O\&M,t} = I \cdot k_{O\&M}. \tag{10}$$

In addition, it is assumed that expenses $C_{O\&M,t}$ increase from year to year according to an exponential function at a rate $r_{O\&M}$:

$$C_{O\&M,t} = C_{O\&M,t_0} \cdot (1 + r_{O\&M})^{t-t_0}, \tag{11}$$

where $C_{O\&M,t_0}$represents the amount of O&M expenses at a reference year t_0, either $t_0 = 1$:

$$C_{O\&M,t_0} = I \cdot k_{O\&M}. \tag{12}$$

Under these conditions, the updated total expenses $CTA_{O\&M}$, associated with O&M works, during the study period are determined with the expression:

$$\boldsymbol{CTA_{O\&M}} = \sum_{t=1}^{T_{sn}} \boldsymbol{C_{O\&M,t}} \cdot (\mathbf{1} + i)^{\boldsymbol{\theta - t}}. \tag{13}$$

The dynamic calculation model (13) can be transformed into a static-equivalent model, with a generic form as follows [11]:

$$\boldsymbol{CTA_{O\&M}} = \boldsymbol{C_{O\&M,0}} \cdot \boldsymbol{\bar{T}_{T,x_1}}, \tag{14}$$

where: $C_{O\&M,0}$ represents the value of the annual O&M calculation expenses:

$$C_{O\&M,0} = C_{O\&M,t_0} \cdot (1 + r_{O\&M})^{-t_0}, \tag{15}$$

$C_{O\&M,t_0}$- the amount of annual O&M expenses in the reference year t_0;
$\bar{T}_{T,x_1}$- equivalent/recalculated duration of the study period:

$$\bar{T}_{T,x_1} = \sum_{t=1}^{T} \left(\frac{1+i}{1+r_{O\&M}}\right)^{-t} = \sum_{t=1}^{T} (1 + x_1)^{-t}. \tag{16}$$

In expression (16) $\bar{T}_{T,x_1}$represents the updated/recalculated duration of the study period, and *i* – the discount rate.

The duration value $\boldsymbol{\bar{T}_{T,x_1}}$can be determined by a direct calculation:

$$\bar{T}_{T,x_1} = [1 - (1 + x_1)^{-T_{sn}}]/x_1, \tag{17}$$

where $\boldsymbol{x_1}$ represents an equivalent discount rate, obtained with the formula:

$$x_1 = (1 + i)/(1 + r_{O\&M}) - 1. \tag{18}$$

Thus, for the O&M expenses associated with the two types of heat sources it can be written:

for Cs - $$CTA_{O\&M}^{Cs} = C_{O\&M,0}^{Cs} \cdot \bar{T}_{T,x_1}^{Cs} = I^{Cs} \cdot k_{O\&M}^{Cs} \cdot \left(1 + r_{O\&M}^{Cs}\right)^{-t_0} \cdot \bar{T}_{T,x_1}^{Cs}, \tag{19}$$

for Cz - $$CTA_{O\&M}^{Cz} = C_{O\&M,0}^{Cz} \cdot \bar{T}_{T,x_1}^{Cz} = I^{Cz} \cdot k_{O\&M}^{Cz} \cdot \left(1 + r_{O\&M}^{Cz}\right)^{-t_0} \cdot \bar{T}_{T,x_1}^{Cz}. \quad (20)$$

If in (19)-(20) the investments $I^{Cs}$and $I^{Cz}$are substituted with the expressions (5) and (6), we get:

$$CTA_{O\&M}^{Cs} = q_M^{sarc} \cdot x \cdot i_s^{Cg} \cdot k_{O\&M}^{Cs} \cdot \left(1 + r_{O\&M}^{Cs}\right)^{-t_0} \cdot \bar{T}_{T,x_1}^{Cs}, \quad (21)$$

and $$CTA_{O\&M}^{Cz} = q_M^{sarc} \cdot (1 - x) \cdot i_s^{Cz} \cdot k_{O\&M}^{Cz} \cdot \left(1 + r_{O\&M}^{Cz}\right)^{-t_0} \cdot \bar{T}_{T,x_1}^{Cz}. \quad (22)$$

Here we will note $a_1 = k_{O\&M}^{Cs} \cdot \left(1 + r_{O\&M}^{Cs}\right)^{-t_0} \cdot \bar{T}_{T,x_1}^{Cs}$: (2.3)

$$a_2 = k_{O\&M}^{Cz} \cdot \left(1 + r_{O\&M}^{Cz}\right)^{-t_0} \cdot \bar{T}_{T,x_1}^{Cz} \quad (24)$$

and respectively - $$A_{O\&M} = a_1 \cdot i_s^{Cs} - a_2 \cdot i_s^{Cz}, \quad (25)$$

so that for the total updated O&M expenses for the two sources, finally, it can be written:

$$\boldsymbol{CTA_{O\&M}^{Cs+Cz} = q_M^{sarc}} \cdot [\boldsymbol{x} \cdot \boldsymbol{A_{O\&M}} + \boldsymbol{a_2} \cdot \boldsymbol{i_s^{Cz}}]. \quad (26)$$

Fuel expenses

The expenses for the fuel consumed in the boiler installations during the study period are presented by means of the static-equivalent model in its general form:

$$\boldsymbol{CTA_{comb}^{Cz} = C_{comb,0}^{Cz} \cdot \bar{T}_{T,x_2}}, \quad (27)$$

where: $C_{comb,0}^{Cz}$ represents the value of annual calculation expenses, related to fuel consumption - (28)

$$C_{comb,0}^{Cz} = B_{comb,t_0} \cdot c_{comb,t_0} \cdot (1 + r_{comb})^{-t_0};$$

B_{comb,t_0}- fuel consumption (expressed in energy units) in the reference year t_0 -

$$B_{comb,t_0} = Q_{t_0}^{Cz}/\eta_{med}; \quad (29)$$

$\bar{T}_{T,x_2}$- the recalculated duration of the study period, determined at the rate x_2:

$$\bar{T}_{T,x_2} = [1 - (1 + x_2)^{-T}]/x_2; \quad (30)$$

x_2- an equivalent discount rate:

$$x_2 = (1 + i)/(1 + r_{comb}) - 1; \quad (31)$$

c_{comb,t_0}- fuel price (per unit of included energy) in the year t_0;

r_{comb} - the annual growth rate of the fuel price over the years of the study period;

η_{med} - the annual average value of boiler efficiency.

After some substitutions, formula (27) takes the form:

$$\boldsymbol{CTA^{Cz}_{comb}} = Q^{Cz} \cdot \boldsymbol{k^{Cz}_{comb}}\,, \tag{32}$$

where: Q^{Cz}represents the volume of heat produced in boiler installations in the year t_0, and k^{Cz}_{comb}- a generalized parameter, related to the cost of fuel consumed during the study period:

$$k^{Cz}_{comb} = c_{comb,t_0} \cdot (1 + r^{Cz}_{comb})^{-t_0} \cdot \bar{T}^{Cz}_{T,x_3} / \eta^{Cz}_{med}. \tag{33}$$

In formula (32) the annual heat production Q^{Cz} is to be expressed expressly by the variable *x* and some known initial data. For this purpose Q^{Cz}we will write:

$$Q^{Cz} = q^{Cz}_{nom} \cdot T^{Cz}_{u}(x), \tag{34}$$

or

$$Q^{Cz} = q^{sarc}_{M} \cdot (1 - x) \cdot T^{Cz}_{u}(x), \tag{35}$$

where $T^{Cz}_{u}(x)$ represents the duration of use of the nominal capacity of the boilers, which depends on the variable *x*.

For the duration $T^{Cz}_{u}(x)$ of the peak area of the load curve, covered by boilers, a simple calculation expression has been obtained in [12]:

$$T^{Cz}_{u,v}(x) = \tau_x \cdot f_{v\hat{a}rf}\,, \tag{36}$$

where: τ_x represents the calendar duration of the peak base:

$\tau_x = \tau_0 \cdot \tau_{an} \cdot (1 - x)^{1/\beta}$ and $\tau_0 = (1 - q_{min*})^{-1/\beta}$;

$f_{v\hat{a}rf}$ - the load factor of the peak area of the curve:

$$f_{v\hat{a}rf} = \beta / (1 + \beta); \tag{37}$$

β - the exponent of the power function in the analytical description of the load curve;

q_{min*} - the minimum annual burden of DHS, in r.u.:

$$q_{min*} = q^{sarc}_{min} / q^{sarc}_{M}. \tag{38}$$

Finally, expression (32) is transformed into formula (39) from which the derivative on *x* can be taken:

$$CTA^{Cz}_{comb} = q^{sarc}_{M} \cdot (1 - x)^{\frac{1+\beta}{\beta}} \cdot \frac{\beta}{(1 + \beta)} \cdot \tau_0 \cdot \tau_{an} \cdot k^{Cz}_{comb}\,. \tag{39}$$

The optimal solution

The optimal value of the solar fraction x_{opt}in covering the maximum annual thermal load of DHS, can be obtained from the optimality condition - $\partial \mathrm{F}/\partial \mathrm{x} = 0$, presented in the form of the sum of the partial derivatives D_j of the three components of the objective function $F(x)$:

$$u(x) = D_I + D_{O\&M} + D_{comb} = 0, \quad (40)$$

where: $D_I = q_M^{sarc} \cdot A_I$, (41) $D_{O\&M} = q_M^{sarc} \cdot A_{O\&M}$ (42)

and $D_{comb} = -q_M^{sarc} \cdot (1-x)^{1/\beta} \cdot \boldsymbol{V},$ (43)

and $A_I = i_S^{Cs} - i_S^{Cz},\ A_{O\&M} = a_1 \cdot i_S^{Cs} - a_2 \cdot i_S^{Cz}andV = \tau_{an} \cdot \tau_0 \cdot k_{comb}^{Cz}$.

Taking into account expressions (41)-(43), equation (40) can be rewritten:

$$u(x) = A_I + A_{O\&M} - (1-x)^{\frac{1}{\beta}} \cdot \boldsymbol{V} = 0. \quad (44)$$

If we write - $\Phi = \boldsymbol{A_I} + \boldsymbol{A_{O\&M}}$(45), equation (44) takes the form -

$$\Phi - (1-x)^{\frac{1}{\beta}} \cdot V = 0 \text{ or } (\boldsymbol{1} - \boldsymbol{x})^{\boldsymbol{1/\beta}} = \Phi/V, \quad (46)$$

from which, in the end, the expression for calculating the optimal solar fraction results x_{opt} -

$$x_{\boldsymbol{opt}}^{Cs} = \boldsymbol{1} - (\Phi/\boldsymbol{V})^{\boldsymbol{\beta}}. \quad (47)$$

It should be noted that in (47) - the component$(\Phi/\boldsymbol{V})^{\boldsymbol{\beta}}$ represents nothing more than the optimal share of the reference source (hot water boilers) - $x_{\boldsymbol{opt}}^{Cz} = (\Phi/\boldsymbol{V})^{\boldsymbol{\beta}}$, since -$x^{Cs} + x^{Cz} = 1$.

Determination of the solar fraction X , in *the annual heat production* of DHS

The size x_*, according to (1), expresses the share of the heat generation power in DHS, related to the solar collectors, and respectively $(1 - x_*)$ represents the share of the power generation of the reference source. On the ranked curve of the thermal load of the system (fig. 7) the size/level x_* it delineates the core from the peak area and the respective energy production technologies.

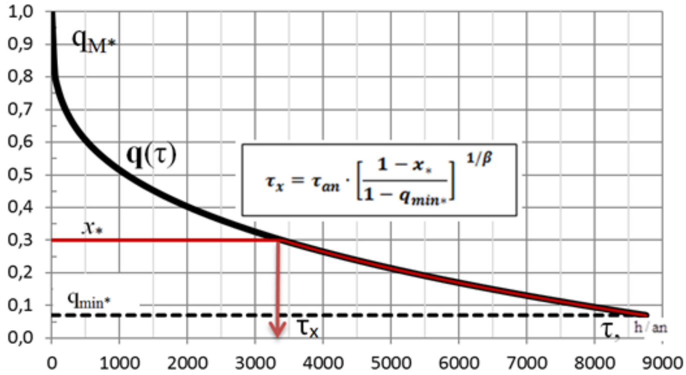

Figure 7. Ranked annual heat load curve: determination of the area of the base and peak surfaces for a value of x of sharing

It should be noted that for any value of x_*, $\{x_* = 0.1\}$, including the optimal value $\boldsymbol{x_{opt}^{Sc}}$, the surface area of the base zone and the peak can be easily determined, which is equivalent to knowing the *Tu* durations of the sources in the base zones and top, *Tu* $=\{T_u^b, T_u^v\}$; in particular for $T_u^{vârf}$ we can write [13] :

$$T_u^{vârf} = \tau_x \cdot f, \tag{48}$$

where $$\tau_x = \tau_{an} \cdot [(1 - x_*)/(1 - q_{min*})]^{1/\beta} \tag{49}$$

and $$f = \beta/(1 + \beta). \tag{50}$$

From the DHS energy balance equation for the source in the base area, the duration calculation formula results $T_u^{bază}$:

$$T_u^{bază} = \left[T_u^{SACET} - T_u^{vârf} \cdot (1 - x_*)\right]/x_* , \tag{51}$$

and finally -

$$T_u^{bază} = \left[T_u^{SACET} - \tau_{an} \cdot f \cdot (1 - x_*)^{1/f}/(1 - q_{min*})^{1/\beta}\right]/x_*. \tag{52}$$

Once the optimal value of the fraction (power) $\boldsymbol{x_{opt}^{Cs}}$ will be known, the optimal value of the fraction (energy) can easily be determined $\boldsymbol{X_{opt}^{Cs}}$.

The expression for calculating the *optimal solar fraction in the annual heat production* $\boldsymbol{X_{opt}^{Cs}}$, resulting from (2), is:

$$X_{opt}^{Cs} = x_{opt}^{Cs} \cdot T_u^{Cs}/T_u^{SACET} , \tag{53}$$

where T_u^{SACET} it represents the duration of use of the total nominal power of the sources, integrated in DHS,
and T_u^{Cs}- duration of use of the nominal power of Cs.

5. Numerical example

We will consider a residential urban area, supplied with thermal energy through a heating system, which integrates two sources of thermal energy: a solar thermal plant and a thermal plant, equipped with hot water boilers.

In the Republic of Moldova, the average annual intensity of the specific solar radiation is 1211 kWh/m^2 - in the North area, 1300 - in the Center, and 1378 kWh/m ^{2} - in the South area. The average daily intensity in December is about 1.0 kWh/m^2 - in the North and 1.4 kWh/m^2 - in the South, while in June it is about 1.0 kWh/m^2 /day - in the North and 1.4 kWh/m^2 /day - in the South. The average annual value of the specific solar intensity in Moldova is 1300 kWh/m^2. In the country, there are about 2060 hours of sunshine in the North and about 2360 hours - in the South, which is below the EU average.

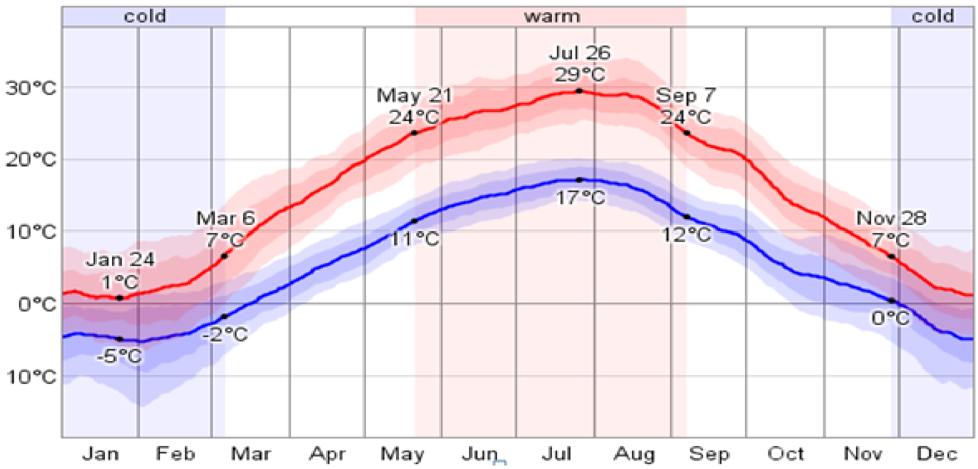

Figure 8. Moldova: average daily maximum and minimum temperatures

The annual ranked curve of the thermal load of the heating system is shown in figure 9. The maximum annual thermal load is 31.7 MW$_t$, minimum load – 2.26 MW$_t$, the annual heat consumption is 83,800 MWh.

Solar collector system

We will admit the use of a field of solar collectors, with a total area of 10,000 m^2; the characteristics of Cs are shown in tab. 1. The specific investment in the collector system depends on the size of the collector field.

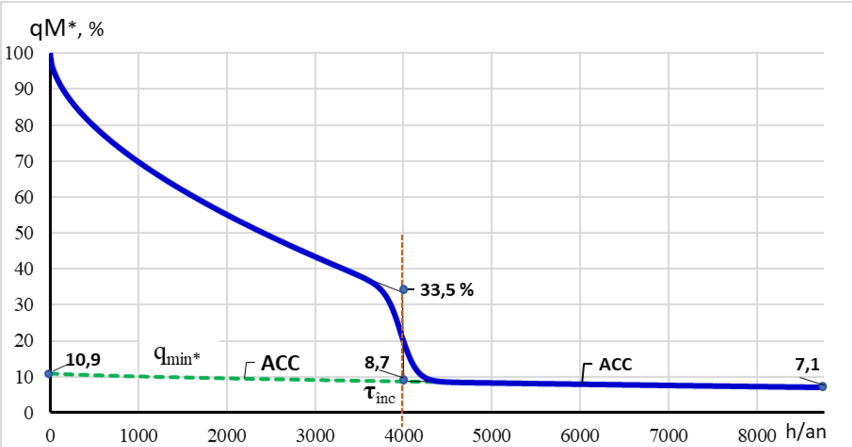

Figure 9. The annual ranked curve of the thermal load of the heating system

Table 1. **Characteristics of solar collectors**

Parameter	m.u.	The value
Collector area	m^2	10000
Specific heat production	$kWh/(m^2$ year)	585
The installed capacity of the collector field	MW	7.00
Average annual efficiency	%	43
Lifetime period	years	30.0
Specific rated capacity	kW_t/m^2	0.7
Annual duration of use of the nominal capacity	h/year	835.7
The specific investment	€/m^2	187.0
Annual O&M cost	€/m^2	0.04
Annual O&M cost – the entire solar block	r.u. from I_{all}	0.085

The heat accumulators in DHS allow the use of the heat stored in the summer to cover the thermal load peaks, as well as to supply the heat in the cold period of the year, which leads to a fuel economy, due to the reduction of the operating time of the peak generators (CAF) and optimal loading of energy production systems.

In heating systems with a solar fraction of up to 15-20%, it is required to have short-term heat accumulators, usually made of metal or reinforced concrete; at higher elevations of the solar fraction - seasonal accumulators arranged underground are used.

The heat storage tank (RezMe - metal tank) serves for daytime storage. The estimated cost of RezMe is about 135 €/m^3 [14]. When sizing

RezMe, the rule is often applied: 0.2 m^3 storage volume for every m^2 solar collector area; therefore, a solar thermal plant with a collection surface of approx. 10,000 m^2 requires the use of an accumulator for daytime storage with a volume of approx. 2,000 m^3 - in one or more storage units.

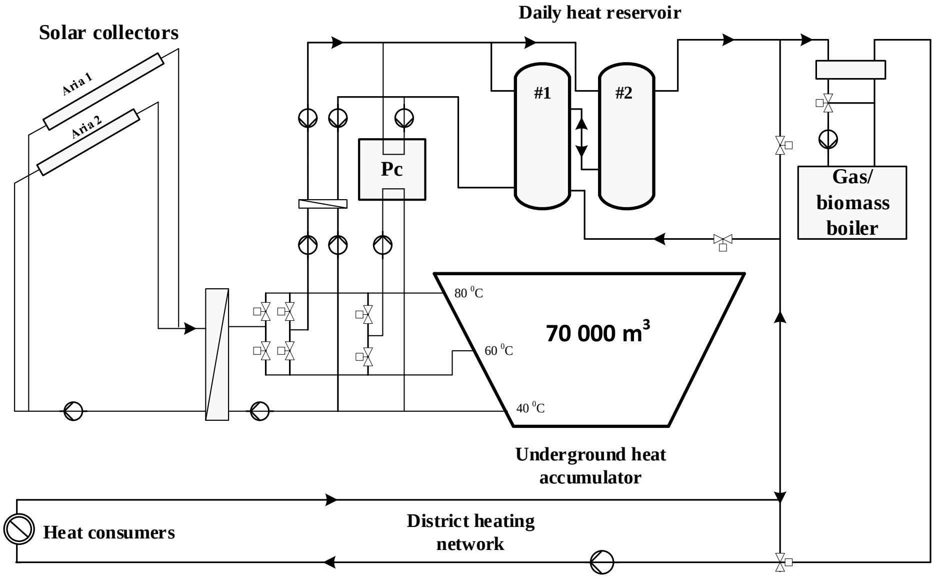

Figure 10. Integration scheme of solar collectors and biomass boilers in a heating system

The seasonal heat storage (eng.: UTES, underground thermal energy storage) serves to store heat during the summer for the purpose of its long-term use up to 4-6 months. It can be connected to DHS directly or indirectly - by means of a heat pump, used to discharge UTES. Battery sizing is done by applying the optimal value of the ratio between UTES volume and Cs surface. In the given case - $R_{V/A}$ = 2.4; for A = 10 000 m^2, V = 24 000 m^3 results.

Table 2. **Characteristics of heat storages**

Parameter	m.u.	The value	
		UTES	RezMe
The ratio of volume accumulator and collector surface	m^3/m^2	2.4	0.2
The storage volume	m^3	24000	2000
The level of losses in the accumulator	%	30	20
The specific investment	€/m^3	61.9	135
Temperature range	^{0}C	20-90	35-85

The capacity of the seasonal storage can be estimated according to the expression –

$$Q_{UTES} = m \cdot c_p \cdot \Delta T = V \cdot \varrho \cdot c_p \cdot \Delta T = 980 \cdot 24000 \cdot 4187 \cdot 50 \cdot 10^{-6} = 1368 \text{ MWh},$$

where: Q_{UTES} represents the amount of stored heat,
V - the volume of the UTES accumulator 24000 m^3,
ϱ - water density 980 kg/m^3,
c_p - heat capacity of water 4186.8 J/(kg·K),
ΔT - the temperature difference between the final and initial state of the storage medium 50 ^{0}C.

The heat pump (Pc) participates in the discharge of the UTES storage, ensuring the desired temperature regime when the heating agent enters the heating network. The use of PC leads to lower storage temperatures throughout the year, which reduces the level of heat loss.

Table 3. **Technical and economic characteristics of heat pumps**

Parameter	um	The value
Installed capacity	MW	5.1
Number of units and unit capacity	MW	3x7, 2x2
Coefficient of Performance (COP)	r.u.	3.0
The specific investment	thousand €/MW	263.5
Annual O&M cost	% of I	30

Hot water boilers

For the boiler plant, two alternative options are considered:

(i) natural gas boilers vs.

(ii) wood biomass boilers (wood shavings, chips, etc.).

Table 4. **Technical and economic characteristics of boilers [15]**

Parameter	m.u.	The value	
		Cz - gases	Cz - biomass
Installed capacity	MW	1-25	1-25
Number of units and unit capacity	MW	3x7, 2x2	3x7, 2x2
Efficiency	%	90.0	75.0
Standard life span	years	20.0	20.0
Duration of use of the installed capacity	years	2000	2000
The specific investment	thousand Euro/MW	60	540
Annual O&M cost	% of I	1,2	2.75

Table 5. **General initial data**

Parameter	m.u.	The value
Discount rate	r.u.	0.1
Annual duration	h/year	8400
Study period	years	20
Discounted study period	years	8.51
Fuel cost		
- natural gases	Euro/MWh	37,28
- wood chips	Euro/MWh	20.0

Numerical calculation

Next, we will determine what is the optimal solar fraction in the hypothesis of realizing the thermal plant as -

(i) natural gas hot water boiler plant and

(ii) plant with solid biomass boilers (chips, wood chips).

Initially we will calculate the optimal fraction (power) $\boldsymbol{x_{opt}}$, and later also the fraction (energy) $\boldsymbol{X_{opt}}$.

The calculation of the optimal fraction (power) for solar collectors is carried out with the formula:

$$\boldsymbol{x_{opt}} = \mathbf{1} - (\Phi/V)^{\boldsymbol{\beta}}.$$

We will start by determining ***the parameter Φ*** based on formula (45):

$$\Phi = A_I + A_{O\&M} = 654.5 + 784.5 = 1{,}439.0 \text{ €/kW},$$

where: $A_I = i_s^{Cs} - i_s^{Cz} = 714.5 - 60.0 = 654.5$ €/kW,
where i_s^{Cs} and i_s^{Cz} represent the specific investments of energy sources[7];
and $A_{O\&M} = a_1 \cdot i_s^{Cs} - a_2 \cdot i_s^{Cz} = 1.112 \cdot 714.5 - 0.157 \cdot 60 = 784.5$ €/kW,
where: coefficients a_1 and a_2 have the values:

$$a_1 = k_{O\&M}^{Cs} \cdot \left(1 + r_{O\&M}^{Cs}\right)^{-t_0} \cdot \bar{T}_{T,x_1}^{Cs} = 0.085 \cdot (1+0.06)^{-1} \cdot 13.87 = 1.112;$$

$$a_2 = k_{O\&M}^{Cz} \cdot \left(1 + r_{O\&M}^{Cz}\right)^{-t_0} \cdot \bar{T}_{T,x_1}^{Cz} = 0.012 \cdot (1+0.06)^{-1} \cdot 13.87 = 0.157;$$

$k_{O\&M}$ - the share of O&M expenses from the total investment value, by energy sources;

$\bar{T}_{T,x_3}$ - recalculated duration of the study period:

$$\bar{T}_{T,x_3} = [1 - (1 + x_3)^{-T}]/x_3 = [1-(1+0.0377)^{-20}]/0.0377 = 13.87 \text{ years};$$

[7] Solar collectors reflect the entire solar block.

x_1 - synthetic discount rate:

$x_1 = (1+i)/(1+r_{O\&M}) - 1$ = (1+0.1)/(1+0.06) – 1 = 0.0377;

$r_{O\&M}$ - ***growth rate of O&M expenses.***

The parameter V is calculated with the formula:

$V = \tau_{an} \cdot \tau_0 \cdot k_{comb}^{Cz}$ = 8400 · 1,233 · 0,587 = 6 075,3 €/kW,

where: τ_{an} represents the annual duration of operation;

τ_0 - a coefficient:

$\tau_0 = (1 - q_{min*})^{-1/\beta}$ = (1-0.072) $^{-1/0.35}$ = 1.233;

q_{min*} - relative annual minimum heat load, $q_{min*} = 0.0717$;

β - the exponent of the power function in the analytical description of the load curve, $\beta = 0.355$;

k_{comb}^{Cz} - a generalized parameter, related to the cost of fuel consumed during the study period:

$k_{comb}^{Cz} = c_{comb,t_0} \cdot (1 + r_{comb}^{Cz})^{-t_0} \cdot \bar{T}_{T,x_3}^{Cz} / \eta_{med}^{Cz}$= = 0.0373 · (1+0.07)$^{-1}$ · 15.15/0.9 = 0.587;

c_{comb,t_0} - fuel price (per unit of embedded energy) in the reference year t_0;

η_{med}^{Cz} - the global average annual efficiency of the boilers;

$\bar{T}_{T,x_3}$ - recalculated duration of the study period:

$\bar{T}_{T,x_3} = [1 - (1 + x_3)^{-T}]/x_3$ = 15.15 years;

x_3 - synthetic discount rate:

$x_3 = (1+i)/(1+r_{comb}) - 1$= (1+0.1)/(1+0.07) – 1 = 0.028;

r_{comb} - the annual fuel price increase rate over the years of the study period.

Finally, the optimal value of the solar fraction –

$$x_{opt}^{Cs} = \mathbf{1} - (\Phi/V)^{\boldsymbol{\beta}} = 1\text{-}(1439.0/6075.3)^{0.355} = 0.40 \text{ r.u.},$$

and respectively of the share of the boilers:

$$x_{opt}^{Cz} = (\Phi/V)^{\boldsymbol{\beta}} = (1439.0/6075.3)^{0.355} = 0.60.$$

Thus, with the DHS characteristics presented above, assuming the use of natural gas hot water boilers, the optimal solar fraction (power) is 40%, respectively the share of boilers – 60%.

It is of interest to also determine the solar fraction (energy) for this situation. The calculations made according to formula (53) lead to the optimal value of the solar fraction - X_{opt}^{Cs}= 0.85, respectively of the optimal share of the boilers - 0.15.

If solid biomass boilers are used instead of natural gas boilers, the numerical calculation results in the following values:

- fraction (power) - x_{opt}^{Cs}= 0.38 and x_{opt}^{Cz}= 0.62,
- fraction (energy) - X_{opt}^{Cs}= 0.84 and X_{opt}^{Cz}= 0.16.

6. Discussions

For about two decades, a number of countries around the world have shown increased interest in solar thermal power plants, integrated into district heating systems - as an excellent solution for decarbonizing the thermal energy sector. The European Union is the leader in this field – with Denmark and Germany leading the way.

The optimal share of solar in a heating system, like the general problem of justifying the structure of energy sources in DHS - belongs to the field of optimization.

In this paper, for a slightly simplified approach, where solar thermal technology (solar thermal collectors plus heat storage, plus heat pumps to discharge tanks) is pitted against natural gas or biomass hot water boilers - simple analytical expressions are obtained to calculate the solar fraction in the heat generation power in DHS and respectively the solar fraction in the annual heat production in the system.

7. Conclusion

The European Union and the whole world, under the auspices of the UN, have taken the course towards a deep decarbonisation of human society in order to remove the risks related to climate change.

The active promotion of energy efficiency and renewable energy sources is, from now on, an indispensable duty of all the states of the world. The Republic of Moldova, a small state, will do everything in order not to fall behind in achieving this goal.

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