https://doi.org/10.52326/ic-ecco.2021/CE.02

Comparative Analysis of the PID Algorithm Synthesis at the Object Model with Astatism and Dead Time

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Abstract **-The paper presents the comparative analysis of the synthesis methods of the PID tuning algorithm for the model of the object with first degree astatism and dead time. In the practice of industrial and technological process automation, mathematical models attached to processes are considered models with first degree astatism and dead time. It analyzes the methods that can be applied for tuning algorithms to these types of process models. Systems with dead time transfer elements do not have finite dimensional systemic achievements, but have an infinite number of poly-zeros. In practice, these models are approximated with rational forms known as Pade approximations with minimal and non-minimal phase. The method of tuning the PID controller shall be analyzed using analytical method of maximum degree of stability and method of maximum degree of stability with iterations. In the object model the dead time component is approximated with Pade approximations with minimal phase and for these models the PID algorithm is synthesized according to the method of the maximum degree with iterations. The PID algorithm is synthesized according to the proposed methods for two examples of values of the parameters of the model of the control object with astatism and dead time and the obtained results are analyzed. The advantages of the method of maximum stability with iterations are highlighted.**

Keywords-Object models with astatism and dead time; Pade approximations; PID algorithm, tuning methods; method of maximum degree of stability with iterations

I. INTRODUCTION

When automating different industrial and technological processes, mathematical models attached to processes are considered as models with astatism and dead time described by the transfer function [1-7]:

$$H(s)=\frac{ke^{-ds}}{Ts}, \tag{1}$$

where k is the transfer coefficient, T - integration time constant, d - dead time.

The presence in the automatic system of the dead time transfer element leads to difficult problems in developing control algorithms. Systems with dead time transfer elements do not have finite dimensional systemic achievements - they have an infinite number of poly-zeros. In practice these elements are approximated with rational forms known as Pade approximants [1-3, 5, 7]. Further, for the transfer element with astatism and dead time, non-minimal phase Pade approximations are used and model (1) is presented with the transfer functions of the form:

$$H(s)=\frac{ke^{-ds}}{Ts}\approx\frac{k-\frac{kds}{3}}{Ts(1+\frac{2ds}{3}+\frac{d^2s^2}{6})}=\frac{-b_0s+b_1}{a_0s^3+a_1s^2+a_2s}, \tag{2}$$

$$H(s)=\frac{ke^{-ds}}{Ts}\approx\frac{k-\frac{kds}{2}}{Ts(1+\frac{ds}{2})}=\frac{-b_0s+b_1}{a_0s^2+a_1s}, \tag{3}$$

$$H(s)=\frac{ke^{-ds}}{Ts}\approx\frac{k-\frac{kds}{2}+\frac{kd^2s^2}{12}}{Ts(1+\frac{ds}{2}+\frac{d^2s^2}{12})}=\frac{b_0s^2-b_1s+b_2}{a_0s^3+a_1s^2+a_2s}, \tag{4}$$

where the coefficients in (2)-(4) are expressed by the parameters k, T, d of the object (1) as:

https://doi.org/10.52326/ic-ecco.2021/CE.02

in (2) $b_0 = kd/3,\ b_1 = k,\ a_0 = Td^2/6,\ a_1 = 2Td/3,\ a_2 = T$;
in (3) $b_0 = kd/2,\ b_1 = k,\ a_0 = Td/2,\ a_1 = T$;
in (4) $b_0 = kd^2/12,\ b_1 = kd/2,\ b_2 = k,\ a_0 = Td^2/12,$ $a_1 = Td/2,\ a_2 = T$.

For tuning the parameters of the PID controller to the model of the object with astatism and dead time (1) can be used methods: empirical methods, frequency methods, parametric optimization, etc. [1, 3-7]. By empirical methods the calculations are reduced, but low performances are also obtained. Frequency methods contain a large volume of calculations with graphical representations in the frequency domain. The method widely used to tune PID-type controllers as the Ziegler-Nichols method cannot be applied [1, 2]. The parametric optimization method is performed in MATLAB.

It is proposed to use analytical method of maximum stability (AMSD) and method of maximum stability with iteration (MSDI) [8-11] for tuning the PID controller to the initial model of the control object with astatism and dead time (1) and approximate models (2)-(4).

The procedure for tuning the PID controller to two models of control object model with astatism and dead time with known parameters and to three approximate models with approximations of non-minimum phase Pade is analyzed.

II. PID CONTROLLER TUNING ALGORITHMS

The study it is used the structural block scheme of the automatic system (AS) made up of the model of the object with transfer function $H_P(s)$ and controller with transfer function $H_R(s)$ given in Fig. 1.

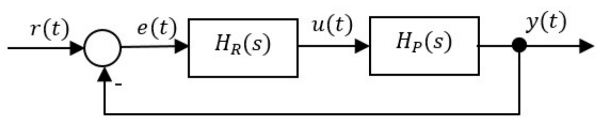

Figure 1. Structural block scheme of the automatic system.

The standard PID tune algorithm is described with the transfer function:

$$H_{PID}(s) = k_p + \frac{k_i}{s} + k_d s = \frac{k_d s^2 + k_p s + k_i}{s}, \qquad (5)$$

where $k_p,\ k_i,\ k_d$ are tuned parameters of the proportional, derivative and integrative component of the PID algorithm.

Applying the analytical method of the maximum degree of stability, the calculation expressions of the PID controller parameters for the models of the control object (1)-(4) in the form [9-11] are presented.

For model (1) the analytical relations are obtained:

$$d^2J^2 - 4dJ + 2 = 0, \qquad (6)$$

$$k_d = \frac{Te^{-dJ}}{2k}(-d^2J^2 + 4dJ - 2) = f_d(J) \approx 0.206T/k, \quad (7)$$

$$k_p = \frac{Te^{-dJ}}{k}(-dJ^2 + 2J) + 2k_dJ = \frac{Te^{-dJ}}{k}(-d^2J^3 + 3dJ^2) = f_p(J) \approx 0.7836T/kd, \qquad (8)$$

$$k_i = -\frac{TJ^2e^{-dJ}}{k} - k_dJ^2 + k_pJ) = \frac{Te^{-dJ}}{2k}(-d^2J^4 + 2dJ^3) = f_i(J) \approx 0.0803T/kd^2, \qquad (9)$$

Solving (6) it is determined the maximum value of the degree of stability $J = 1.268/d$ and in the relations (7)-(9) the expressions are given after which the optimal values of the parameters $k_p,\ k_i,\ k_d$ of the PID controller are calculated.

For model (2) the analytical expressions are obtained:

$$k_d = \frac{-d_0J^5 - d_1J^4 - d_2J^3 - d_3J^2 + d_4J - d_5}{2(b_0J + b_1)^2}, \qquad (10)$$

where $d_0 = 6a_0b_0^3,\ d_1 = 22a_0b_0^2b_1 - 2a_1b_0^3,\ d_2 = 28a_0b_0b_1^2 - 8a_1b_0^2b_1,\ d_3 = 12a_0b_1^3 - 12a_1b_0b_1^2,\ d_4 = 6a_1b_1^3 - 2a_2b_0b_1^2,$ $d_5 = 2a_2b_1^3$,

$$k_p = \frac{d_0J^4 + d_1J^3 - d_2J^2 + d_3J}{(b_0J + b_1)^2} + 2k_dJ, \qquad (11)$$

where $d_0 = 3a_0b_0,\ d_1 = 4a_0b_1 - 2a_1b_0,\ d_2 = 3a_1b_1 - 2a_0b_0,$ $d_3 = 2a_2b_1$,

$$k_i = \frac{-a_0J^4 + a_1J^3 - a_2J^2}{b_0J + b_1} - k_dJ^2 + k_pJ. \qquad (12)$$

For model (3) the analytical expressions are obtained:

$$k_d = \frac{-d_0J^4 - d_1J^3 - d_2J^2 + d_3J + d_4}{2(b_0J + b_1)^4}, \qquad (13)$$

where $d_0 = 2a_0b_0^3,\ d_1 = 8a_0b_0^2b_1,\ d_2 = 12a_0b_0b_1^2,$ $d_3 = -6a_0b_1^3 + 2a_1b_0b_1^2,\ d_4 = 2a_1b_1^3$,

$$k_p = \frac{-d_0J^3 + d_1J^2 + d_2J}{(b_0J + b_1)^2} + 2k_dJ, \qquad (14)$$

where $d_0 = 2a_0b_0,\ d_1 = -3a_0b_1 + a_1b_0,\ d_2 = 2a_1b_1$,

$$k_i = \frac{a_0J^3 - a_1J^2}{b_0J + b_1} - k_dJ^2 + k_pJ. \qquad (15)$$

For model (4) the analytical expressions are obtained:

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$$k_d = \frac{-d_0J^8 - d_1J^7 - d_2J^6 - d_3J^5 - d_4J^4 - d_5J^3 - d_6J^2 - d_7J - d_8}{2(b_0J^2 + b_1J + b_2)^4}, \quad (16)$$

where $d_0 = 2a_0b_0^3$, $d_1 = 8a_0b_0^2b_1$, $d_2 = 8a_0b_0^2b_2 + 12a_0b_0b_1^2$, $d_3 = 28a_0b_0b_1b_2 + 2a_1b_0^2b_2 + 6a_0b_1^3 - 2a_1b_0b_1^2 - 2a_2b_0^2b_1$, $d_4 = 18a_0b_0b_2^2 + 22a_0b_1^2b_2 - 4a_1b_0b_1b_2 - 2a_1b_1^3 - 2a_2b_0b_1^2 - 6a_2b_0^2b_2$, $d_5 = 28a_0b_1b_2^2 - 4a_1b_0b_2^2 - 8a_1b_1^2b_2 - 8a_2b_0b_1b_2$, $d_6 = 12a_0b_2^3 - 4a_2b_0b_2^2$, $d_7 = 2a_2b_1b_2^2 - 6a_1b_2^3$, $d_8 = 2a_2b_2^3$,

$$k_p = \frac{d_0J^5 + d_1J^4 + d_2J^3 + d_3J^2 + d_4J}{(b_0J^2 + b_1J + b_2)^4} + 2k_dJ, \quad (17)$$

where $d_0 = 2a_0b_0$, $d_1 = -3a_0b_1 - a_1b_0$, $d_2 = 4a_0b_2 - 2a_1b_1$, $d_3 = -3a_1b_2 + a_2b_1$, $d_4 = 2a_2b_2$,

$$k_i = \frac{-a_0J^4 + a_1J^3 - a_2J^2}{b_0J^2 + b_1J + b_2} - k_dJ^2 + k_pJ. \quad (18)$$

The PID control algorithm is synthesized for the object model (1) with known parameters, using the analytical MSD method and MSD method with iteratations [9-11], and for models (2)-(4) the MSD method with iteratations is used.

PID controller tuned parameters k_p, k_i, k_d are analytical functions of the known parameters of the object and of the unknown degree of stability J of the system: $k_p = f_p(J)$, $k_i = f_i(J)$, $k_d = f_d(J)$. According to the relations (7)-(9), (10)-(12), (13)-(15), (16)-(18) the curves $k_p = f_p(J)$, $k_i = f_i(J)$, $k_d = f_d(J)$ are constructed respectively.

In order to obtain the desired system performance with the PID controller on curves $k_p = f_p(J)$, $k_i = f_i(J)$, $k_d = f_d(J)$ sets of values of the parameters of the PID controller $J_i - k_{pi}$, k_{ii}, k_{di} are chosen and the automatic system is simulated on the computer. The transient responses of the system are raised and the the highest performances of the system are determined.

III. COMPUTER APPLICATIONS AND SIMULATIONS

To verify the proposed method of synthesis of the PID controller in the object model (1), two variants of numerical values of the object parameters are used:

1) $k = 1$, $T = 0.1s$, $d = 0.5s$,

2) $k = 0.5$, $T = 10s$, $d = 2s$.

It is calculated the numerical values of the parameters for models (1)-(4).

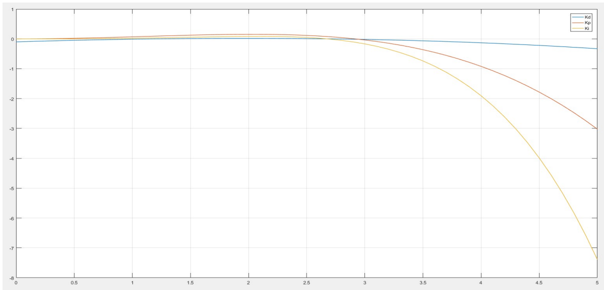
Example 1. For the data $k = 1$, $T = 0.1s$, $d = 0.5s$ of the model the values of the parameters are obtained:

in (2) $b_0 = kd/3 = 1 \cdot 0.5/3 = 0.1667$, $b_1 = k = 1$, $a_0 = Td^2/6 = 0.1 \cdot 0.5^2/6 = 0.0042$, $a_1 = 2Td/3 = 2 \cdot 0.1 \cdot 0.5/3 =$ $a_2 = T = 0.1$;

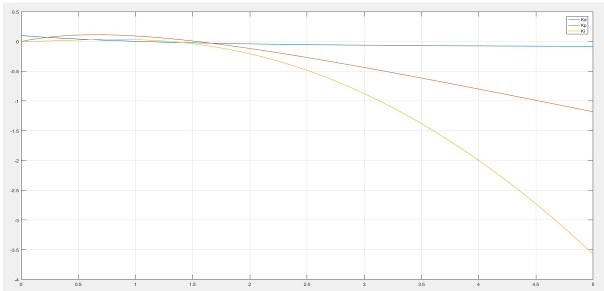
in (3) $b_0 = kd/2 = 1 \cdot 0.5/2 = 0.25$, $b_1 = k = 1$, $a_0 = Td/2 = 0.1 \cdot 0.5/2 = 0.025$, $a_1 = T = 0.1$,

in (4) $b_0 = kd^2/12 = 1 \cdot 0.5^2/12 = 0.0208$, $b_1 = kd/2 = 1 \cdot 0.5/2 = 0.25$, $b_2 = k = 1$, $a_0 = Td^2/12 == 0.1 \cdot 0.5^2/12 = 0.00208$, $a_1 = Td/2 = 0.1 \cdot 0.5/2 = 0.025$, $a_2 = T = 0.1$.

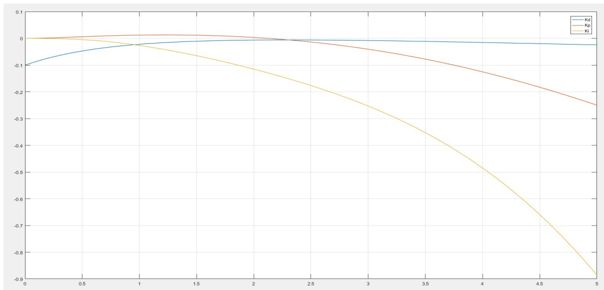
According to the relations (7)-(9), (10)-(12), (13)-(15), (16)-(18) for both variants of numerical data of the parameters of models (2)-(4) were calculated and constructed the respective curves $k_p = f_p(J)$, $k_i = f_i(J)$, $k_d = f_d(J)$ which are given in Fig. 2 (curves: *a*) for model (2), curves *b*) for model (3), curves *c*) for model (4).



a)



b)



c)

Figure 2. Dependencies $k_p = f_p(J)$, $k_i = f_i(J)$, $k_d = f_d(J)$ of the PID algorithm.

Example 2. For the data $k = 0.5$, $T = 10s$, $d = 2s$ of the model the values of the parameters are obtained:

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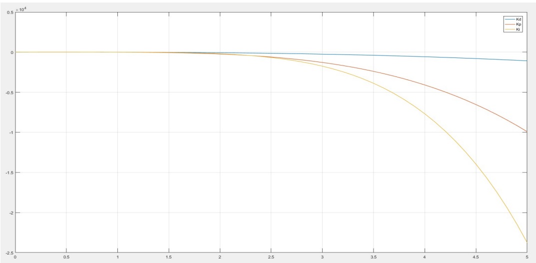
in (2) $b_0 = kd/3 = 0.5 \cdot 2/3 = 0.3333$, $b_1 = k = 0.5$, $a_0 = Td^2/6 = 10 \cdot 2^2/6 = 6.6667$, $a_1 = 2Td/3 = 2 \cdot 10 \cdot 2/3 = 13.3333$, $a_2 = T = 10$;

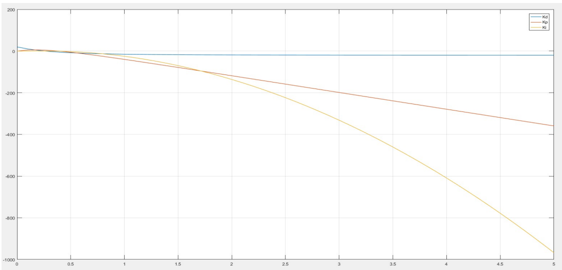
in (3) $b_0 = kd/2 = 0.5 \cdot 2/2 = 0.5$, $b_1 = k = 0.5$, $a_0 = Td/2 = 10 \cdot 2/2 = 10$, $a_1 = T = 10$;

in (4) $b_0 = kd^2/12 = 0.5 \cdot 2^2/12 = 0.1667$, $b_1 = kd/2 = 0.5 \cdot 2/2 = 0.5$, $b_2 = k = 0.5$, $a_0 = Td^2/12 = 10 \cdot 2^2/12 = 3.3333$, $a_1 = Td/2 = 10 \cdot 2/2 = 10$, $a_2 = T = 10$.

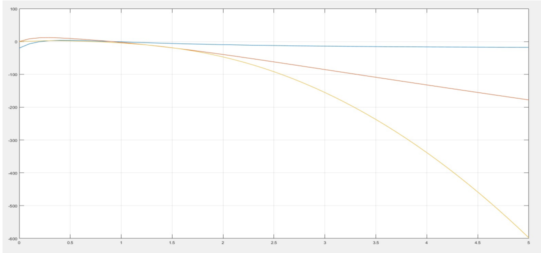
According to the relations (7)-(9), (10)-(12), (13)-(15), (16)-(18) for both variants of numerical data of the parameters of models (2)-(4) were calculated and constructed the respective curves $k_p = f_p(J)$, $k_i = f_i(J)$, $k_d = f_d(J)$ which are given in Fig. 3 (curves: *a*) for model (2), curves *b*) for model (3), curves *c*) for model (4)).



a)



b)



c)

Figure 3. Dependencies $k_p = f_p(J)$, $k_i = f_i(J)$, $k_d = f_d(J)$ of the PID algorithm.

On the curves $k_p = f_p(J)$, $k_i = f_i(J)$, $k_d = f_d(J)$ iterations were chosen and the automatic system was simulated in the MATLAB software package. The simulation scheme is given in Fig. 4.

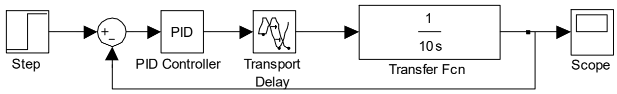


Figure 4. Computer simulation scheme of the automated system.

The transient responses - in Fig. 5, *a*) for the parameters of model 1 and *b*) for the parameters of model 2. The numbering of the curves corresponds to the numbering in Table I.

TABLE I. CONTROLLER PARAMETERS AND PERFORMANCE OF AUTOMATIC SYSTEM

№	Model object	Tuning method	Controller parameters				Performances of the system			
			J	k_p	k_i	k_d	t_c	w	t_r	n
	Data for example 1									
1	(1)	AMSD	2.536	0.1567	0.0839	0.0206	0.93	47.74	3.48	1
2	(1)	MSDI	0.3	0.0151	0.0008	0.0195	11.6	15.91	51.19	1
3	(2)	MSDI	1.3	0.1029	0.0387	0.0039	1.37	37.29	5.52	1
4	(3)	MSDI	0.4	0.0990	0.0185	0.0503	1.98	11.84	9.65	1
5	(4)	MSDI	-	$k_p > 0$	$k_i < 0$	$k_d < 0$	unstable system			
6	(1)	EM	-	0.22	1.0	0.2	unstable system			
7	(1)	PO	-	0.157	0.0023	0.034	1.02	11.00	10.5	1
	Data for example 1									
1	(1)	AMSD	0.634	7.836	1.0495	4.12	3.71	45.00	14.29	1
2	(1)	MSDI	0.9	6.4268	0.4820	3.2399	4.51	24.35	23.59	1
3	(2)	MSDI	0.4	5.0091	0.4451	0.6879	5.61	34.43	23.55	1
4	(3)	MSDI	0.2	5.4815	0.4370	3.1481	5.35	23.83	25.53	1
5	(4)	MSDI	0.7	5.0143	0.0476	2.0929	6.95	3.62	6.95	1
6	(1)	EM	-	11	0.25	0.8	3.63	64.40	25.51	5
7	(1)	PO	-	8.029	0.311	5.0562	1.48	23.7	43	1

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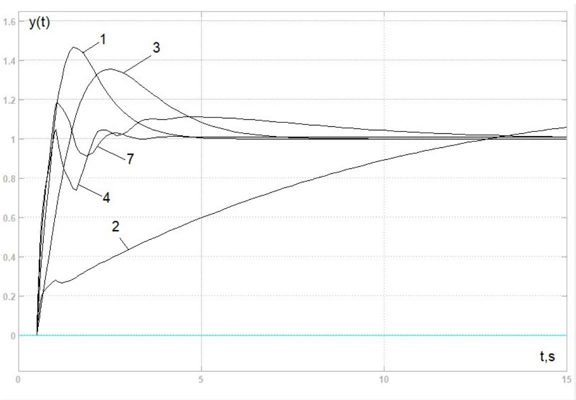
Table I shows the numerical data variant (examples 1 and 2), the object model, the degree of stability and the values of the controller parameters at which the highest system performance was obtained and they are shown in Table 1 in rows 1-5.

In rows 6 (ex. 1) and 6 (ex. 2) of the Table I are presented the values of the tuned parameters according to the empirical method (EM) and the system performances. The transient responses are given in Fig. 5, *a*) - curve 6 and *b*) - curve 6.

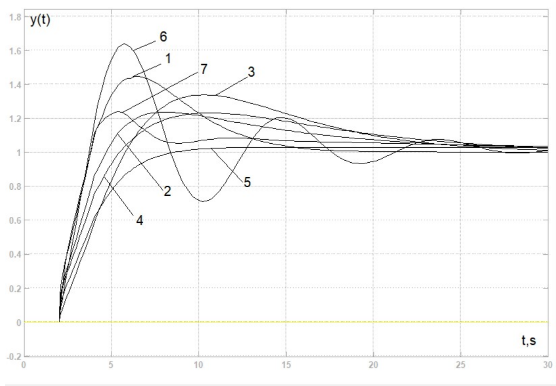
To verify the results obtained when tuning the PID controller to the models of objects (1)-(4) according to the MSD method with iterations, the parametric optimization method (PO) is used and the calculated controller parameters are given in table 1, rows 7 (ex. 1) and 7 (ex. 2). The transient responses are given in Fig. 5, *a*) - curve 7 and *b*) - curve 7 (the numbering of the curves corresponds to the numbering in Table I).

Analyzing the performances of the automatic systems with the PID controller tuned according to the methods indicated at the 5% error of the steady state regime from table 1, the following is found.

The system with PID controller tuned to model (1) according to the analytical MSD method for the data in examples 1 and 2 the performance of the overshoot w is obtained at the value 47.74% and 45% respectively, and the settling time t_r has reduced values at 3.48 and 14.29 respectively.



a)



b)

Figure 5. Transient responses of automatic system with PID controller

For the system with PID controller tuned to the model (1) according to the MSD method with iterations for the data in examples 1 and 2 the performance of the overshoot w was reduced by 3 and 1.85 times respectively and the settling time t_r was increased by 14.71 and 1.65 times respectively compared to the system with the PID controller tuned to model (1) according to the analytical MSD method.

For example 1 with the numerical values of the parameters of the object model $k=1$, $T=0.1$, $d=0.5$ the system with the controller tuned to models (2)-(4) only the system with the controller tuned to model (3) has the performance overshoot w by 4.03 times lower and the settling time t_r is by 2.77 times longer than the same system performance with the controller given in model (1) according to the analytical MSD method. The controller system assigned to model (4) is unstable.

The system with the controller tuned to model (1) with the empirical method is unstable.

For example 2 with the numerical values of the parameters of the object model $k=0.5$, $T=10$, $d=2$ the system with the controller tuned to models (2)-(4) only the system with the controller tuned to model (4) has higher performance the overshoot w 12.43 times and the settling time t_r by 2.06 times compared to the same system performance with the controller tuned to model (1) with the analytical MSD method. The system with controller tuned to model (4) has higher performance overshoot w by 6.73 times and the settling time t_r by 3.39 times compared to the same performance of the system with controller tuned to model (1) according to the MSD with iterations method.

The system with the controller tuned to the model (1) with the empirical method has low performance: the

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overshoot by 2.71 times and the settling time by 1.11 times longer than the respective performances of the system with the controller tuned to the model (1).

IV. CONCLUSIONS

Analyzing the results of the study of the tuning of the PID regulator to the models (1)-(4) from Table I it was found:

- For the automatic system with the PID controller tuned to model (1)-(3) with the parameters from example 1 the performance of the overshoot and the settling time are contradictory. It is recommended to tune the controller to model (3) with the MSD with iterations method.

- For the automatic system with the PID controller tuned to model (1)-(4) with the parameters from example 2 it is recommended to tune the controller to model (4) with the MSD method with iterations.

- Tuning the PID controller to model (1) with the data from example 1 ($d > T$) is more difficult than tuning the controller to the model with the data from example 2 ($d < T$).

ACKNOWLEDGMENT

This work was supported by the project 20.80009.5007.26 „Models, algorithms and technologies for the control, optimization and security of the Cyber-Physical systems”.

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